Chapter 6

Nonlinear Applications

1. Introduction

The device that will be the basis for the nonlinear applications in this chapter is a comparator. A comparator is a device that generates a binary output, high or low depending on whether the difference between 2 input signals is positive or negative. It is clear that the input stage of these comparators is a differential amplifier.

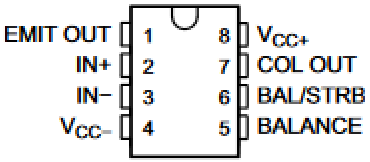
Through an OP AMP can be used as a comparator, it will be slower than a voltage comparator. One of the most important reasons is a compensating capacitor inside the OP AMP to minimize the possibility of oscillations especially in the case of applications where feedback is present. Since the utilization of a voltage comparator usually is about binary outputs (either high or low), feedback will not be needed.

We are going to analyze these applications using one of the most popular comparators, the LM 311, because we can also compare our results to a Multisim simulation whose library includes the LM 311.

2. LM 311

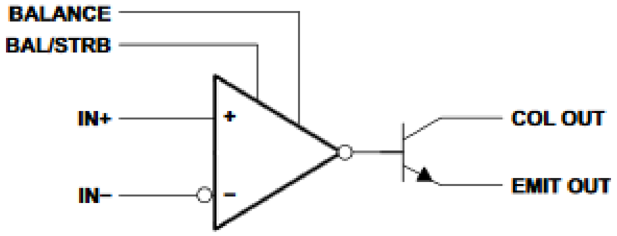
2.1Specifications

The pin diagram is shown below



However, the analysis will be clearer in some cases through the following simplified schematic

Simplified Schematic



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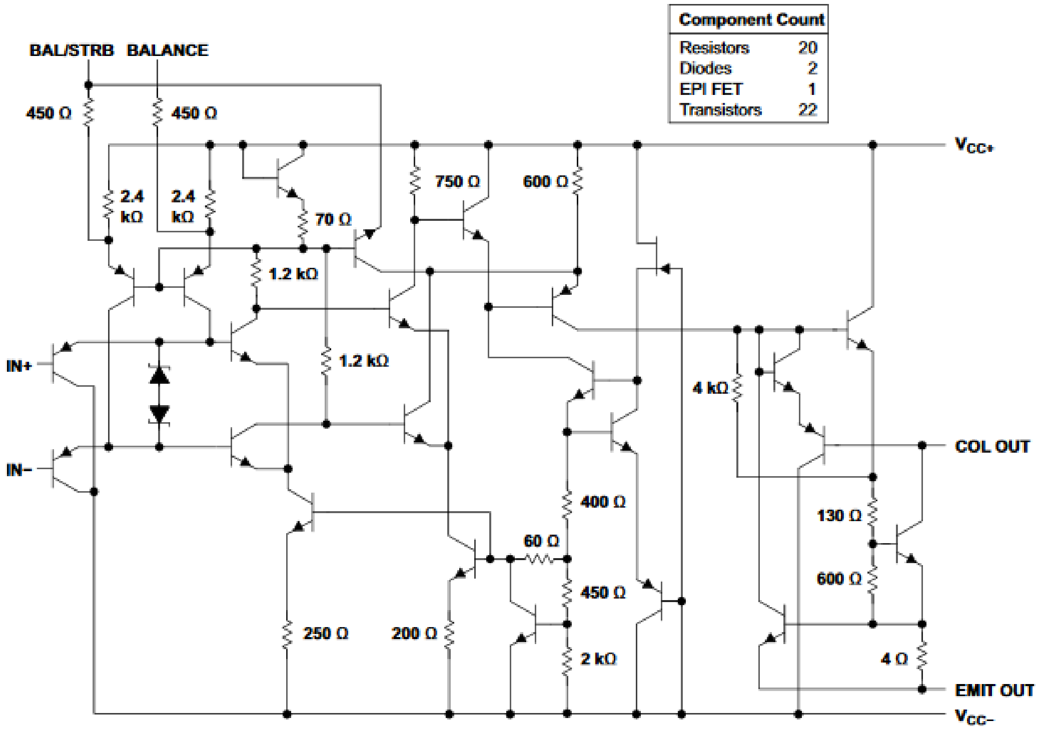
The functional block diagram as given by Texas Instruments for their LM 311 is given below. Note that in some LM311 from other companies, the EMIT OUT is already connected to ground.



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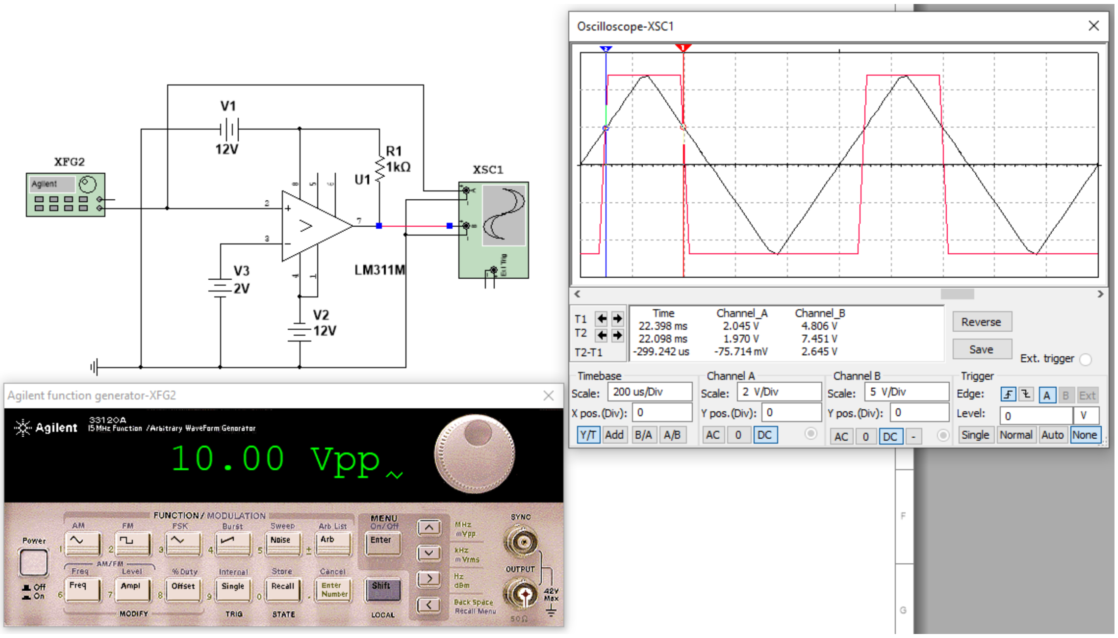
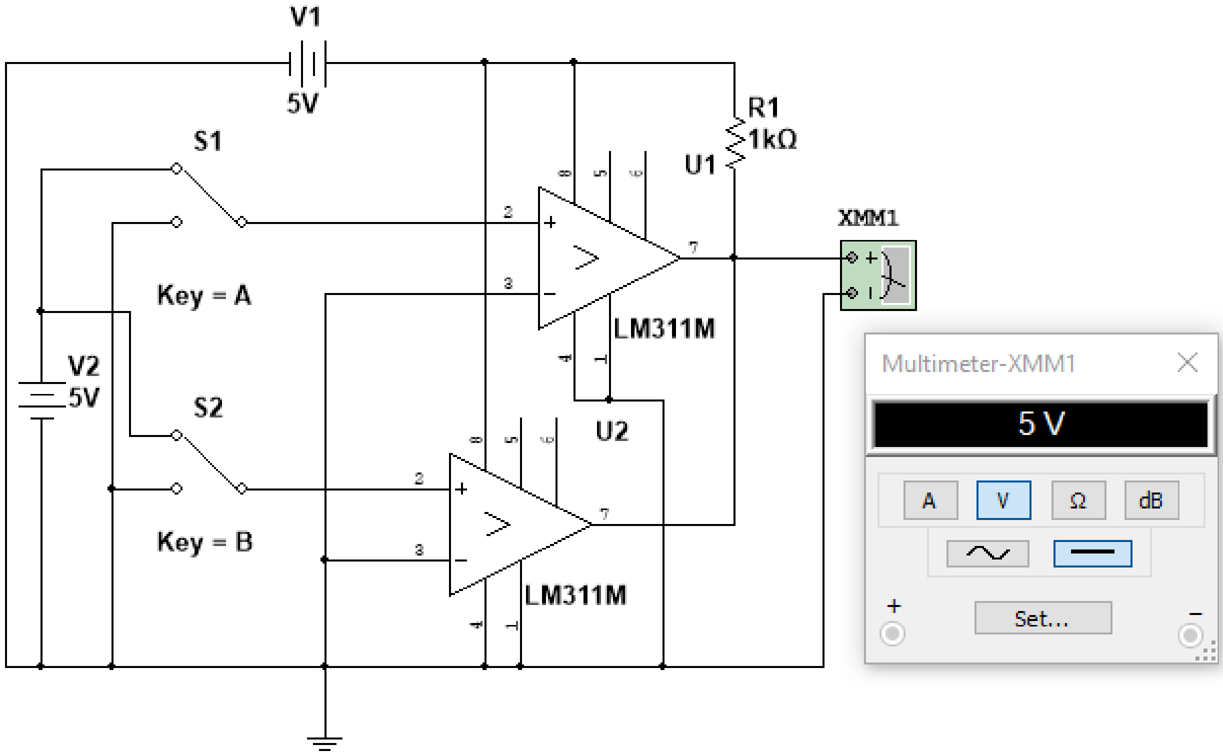
The LM311 is a single high-speed voltage comparator. The transistor from the output stage has floating collector and emitter. We, hence, need to use a pull up or a pull down resistor to work with this comparator. It may be sometimes a convenience to be able to connect the pull up resistor to a power supply different from the rails. The output levels are compatible with most TTL and MOSFET circuits. The outputs can be referenced to ground, V_{CC}, or $-V_{EE}$.



	Time	Channel_A	Channel_B
T1	22.398 ms	2.045 V	4.806 V
T2	22.098 ms	1.970 V	7.451 V
T2-T1	-299.242 us	-75.714 mV	2.645 V

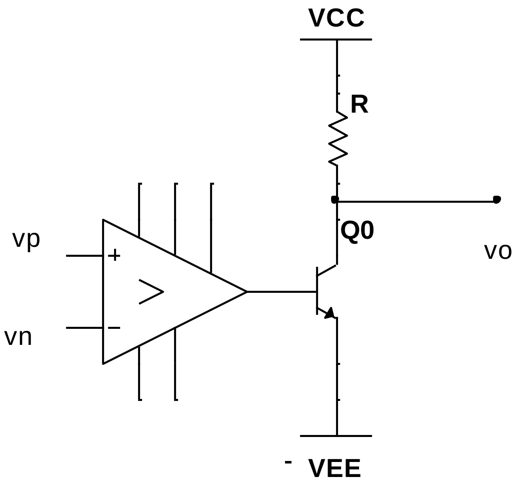
Timebase Scale: 200 us/Div
X pos.(Div): 0
Channel A Scale: 2 V/Div
Y pos.(Div): 0
Channel B Scale: 5 V/Div
Y pos.(Div): 0
Trigger Level: 0 V

The outputs can also be "wire-OR" connected (all open collector outputs must be high (OFF) in order for the output to go high).



5 V

2.2 Output Characteristics under input control



Assume that the transistor Q_0 is the output transistor that is part of the output stage of the LM311, its collector is the pin 7 COL OUT, and its emitter is pin 1 EMIT OUT.
v_p is the noninverting input voltage, and v_n is the inverting input voltage.

$$v_p > v_n \qquad \mathrm{Q_0\ will\ be\ OFF, and\ v_0 = V_{OH} = V_{CC}}$$

$$v_p < v_n \qquad \mathrm{Q_0\ will\ be\ saturated, and\ v_0 = V_{OL}}$$

$$V_{OL} = \begin{Bmatrix} V_{CEsat} - V_{EE} & \text{if EMIT OUT is connected to -}\mathrm{V_{EE}} \\ V_{CEsat} & \text{if EMIT OUT is connected to ground} \end{Bmatrix}$$

In this course, we will assume that V_{CEsat} is approximately 0. It is possible that some comparators may have input voltages not allowed to reach the rail values, and others will have outputs that may not be equal to V_{CC} or $-V_{EE}$ due to internal voltage drops.

3. Comparator Applications

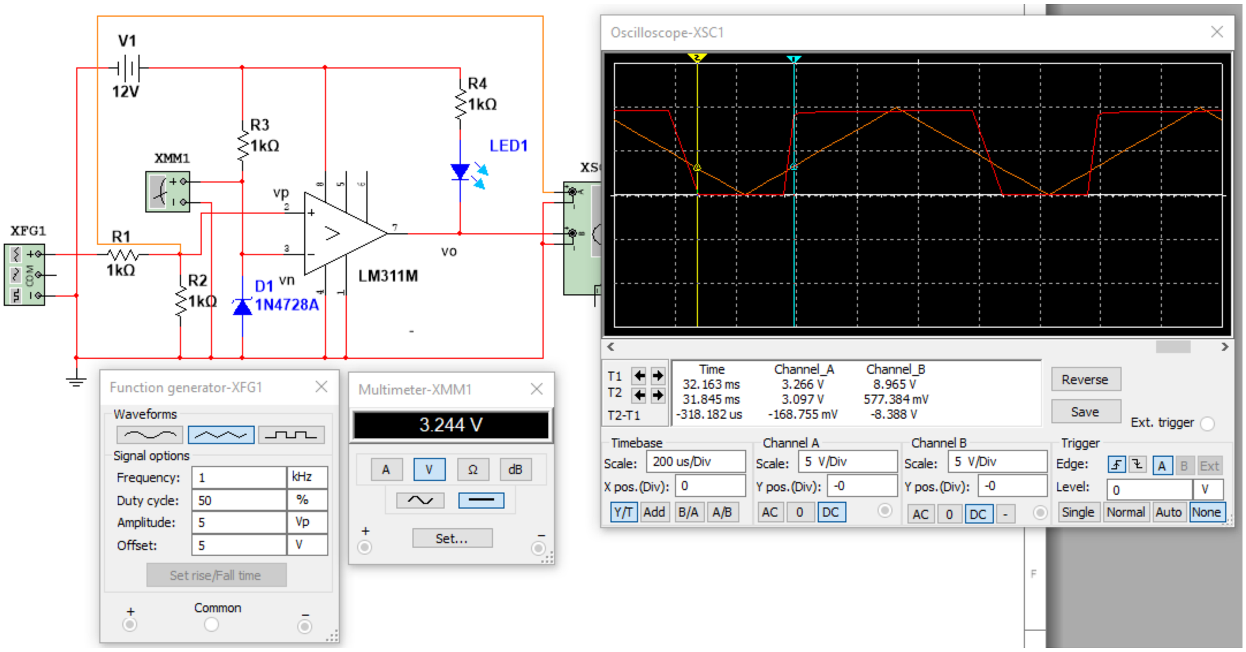
3.1 Level Detector

The level detector shown below is composed of the voltage comparator LM311, a Zener diode whose Zener voltage is nominally 3.3V, an LED, and additional resistors..

R_1 and R_2 act as a voltage divider (in this case, the ratio is 1/2). R_3 is utilized to limit the current into the Zener diode, and help establish a current that is not harmful to the Zener diode while the diode is biased in the reverse region. R_4 helps limit the current into the LED.

In this case, the current flowing through the Zener diode is

$$I_Z = \frac{V_1 - V_Z}{R_3} = \frac{12 - 3.244}{1e3} = 8.76mA$$



Function generator-XFG1

Signal options		
Frequency:	1	kHz
Duty cycle:	50	%
Amplitude:	5	Vp
Offset:	5	V

Multimeter-XMM1: 3.244 V

Oscilloscope-XSC1

	Time	Channel_A	Channel_B
T1	32.163 ms	3.266 V	8.965 V
T2	31.845 ms	3.097 V	577.384 mV
T2-T1	-318.182 us	-168.755 mV	-8.388 V

Timebase Scale: 200 us/Div; X pos.(Div): 0
Channel A Scale: 5 V/Div; Y pos.(Div): -0
Channel B Scale: 5 V/Div; Y pos.(Div): -0
Trigger Level: 0 V

$$Q_0 \text{ is OFF when } v_p > v_n \Rightarrow \text{no current flow} \Rightarrow \text{LED OFF}$$

$$Q_0 \text{ is saturated when } v_p < v_n \Rightarrow \text{a current will flow} \Rightarrow \text{LED ON}$$

$$\text{Since } v_p = \frac{R_2}{R_1 + R_2} v_{in} \; and \; v_n = V_Z = V_{ref}$$

$$\frac{R_2}{R_1 + R_2} V_T = V_{ref} \qquad \text{where } V_T \text{ is the threshold voltage related to } v_{in}$$

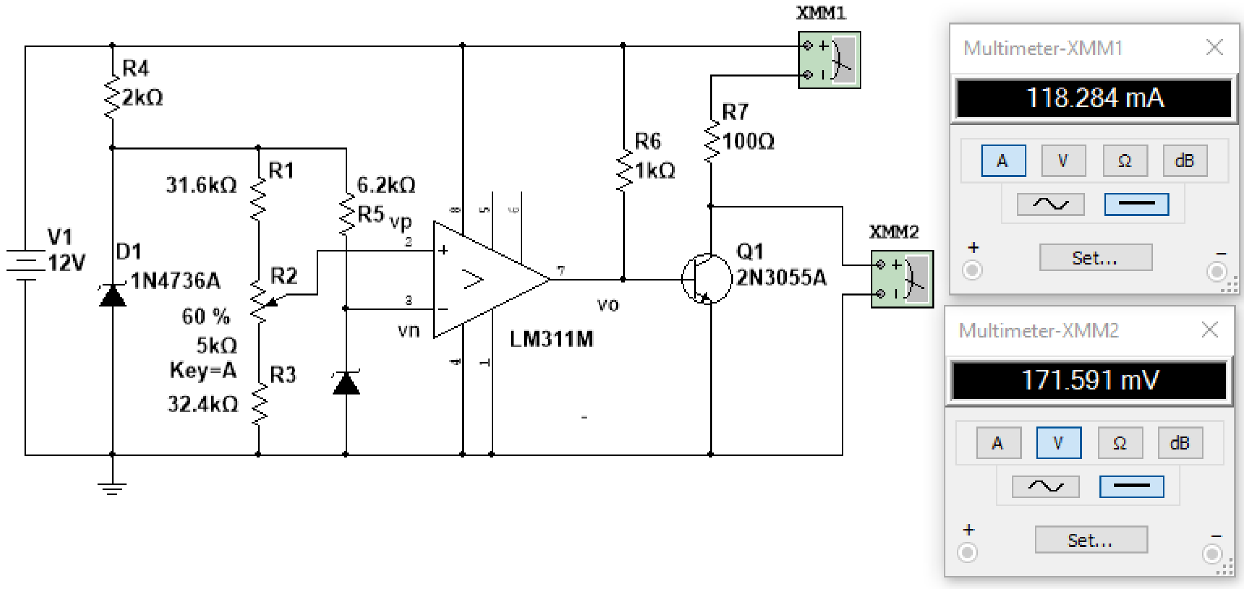
$$V_T = \left(1 + \frac{R_2}{R_1}\right) V_{ref}$$

$$V_T = (1+1)\,3.244 = 6.49V$$

Note that in this case $v_{in} = 2v_p$, and the input signal displayed in Multisim is v_p

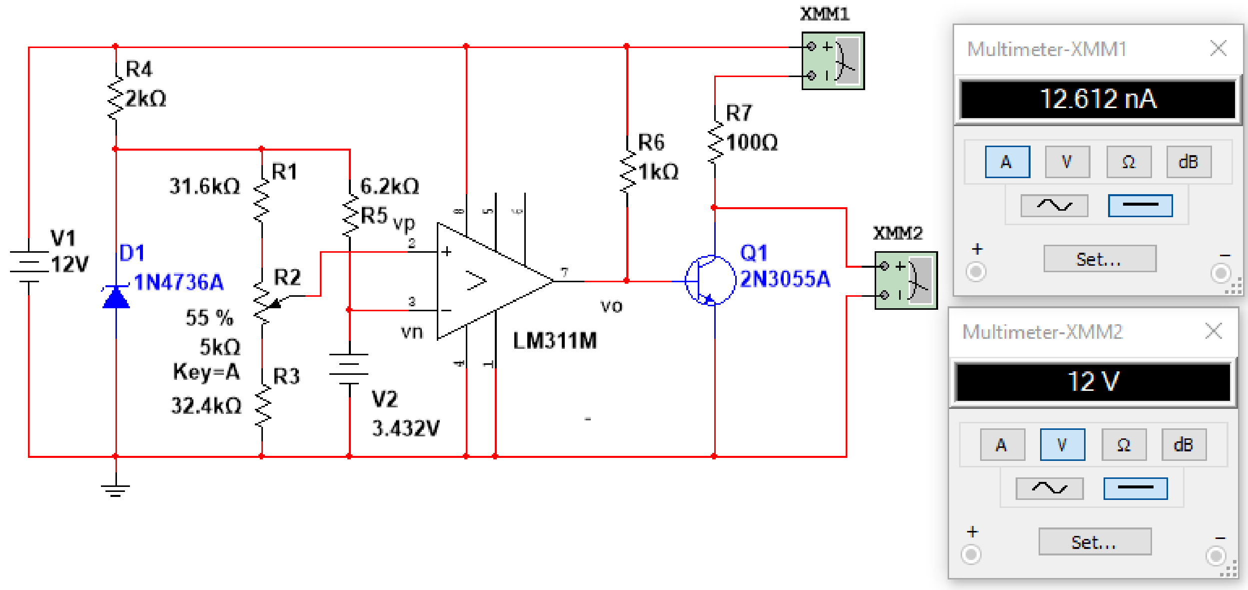
3.2 ON-OFF Control

The circuit shown below describes a device that controls a heating device represented by R_5 such that when the temperature is set through the potentiometer, and if the temperature falls below the set temperature, the transistor is turned ON allowing the heater to raise the temperature. Otherwise, the transistor is turned OFF so that the temperature can go down if the heater is OFF. Let us describe the function of the various stages.



118.284 mA

171.591 mV



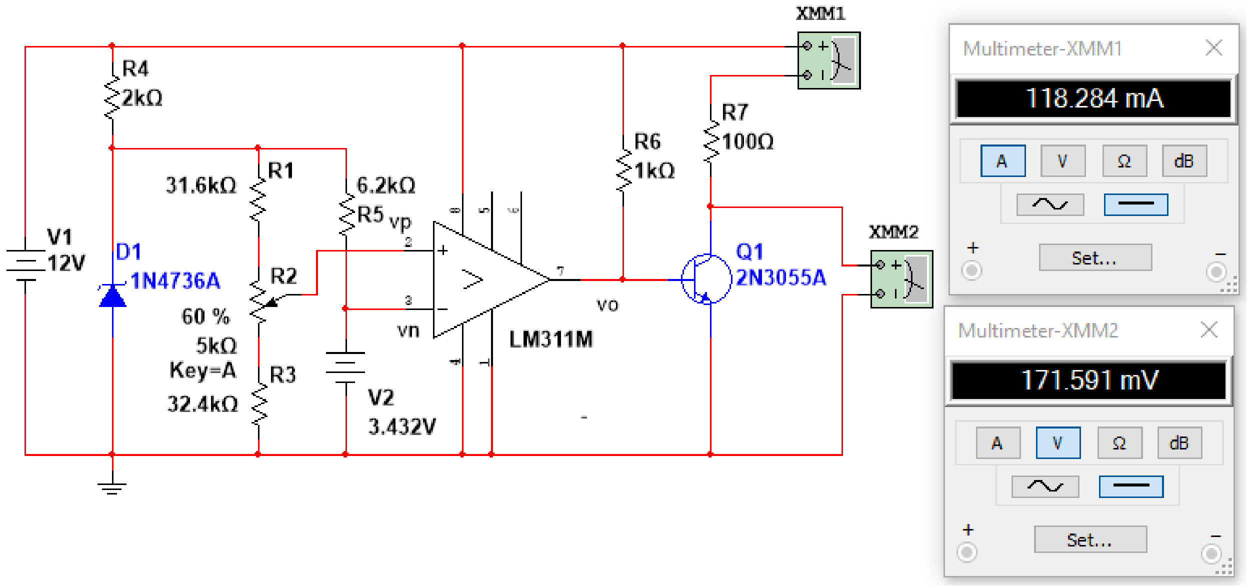
12.612 nA

12 V

Instead of the battery $V_2 = 3.432V$ (in the case of the simulation shown), we are supposed to work with a temperature sensor such as the LM 335, which is a Zener diode where the Zener voltage is directly proportional to the ambient temperature in degrees Kelvin. In this case,

$$V_2 = V_{ZD2} = \frac{T}{100}$$

where T is the temperature in degrees Kelvin.



The combination of D_1 and R_4 allows the circuit to operate as long as the battery V_1 is greater than the Zener voltage of D_1 (6.8V). However, we need a security margin, and we will set the minimum value of the battery to 9V (this includes the voltage drop across R_5, and less than the maximum value that is specified for the LM311.

Let us assume that the operating temperature range is between 50 and 100˚C.

When the arm of the potentiometer is at the extreme top, the voltage v_p will be maximum (simulation shows 100%), corresponding to the highest temperature to work with. When the arm of the potentiometer is at the extreme bottom (simulation shows 0%), the voltage v_p will be minimum, corresponding to the lowest temperature to work with. v_n is the voltage across the Zener diode D2 (temperature-dependent Zener voltage).

The resistors R_1, R_3, and the potentiometer R_2 are chosen to set the extreme temperatures that the system will operate with, and the temperature that will trigger the activity of the heating device.

When the arm of R_2 is all the way up, the temperature under consideration is the largest value in the chosen range, and when the arm is all the way down, the lowest temperature is the one that is being considered. The two resistors and the potentiometer have the same current flowing through them since the current into the ideal comparator is zero (infinite input resistance).

When the arm is all the way up,

$$v_{pmax} = V_{ZD2max} = T_{max}/100$$

and when the arm is all the way down,

$$v_{pmin} = V_{ZD2min} = T_{min}/100$$

If I_{123} is the current through the two resistors and the potentiometer, it can be obtained from

$I_{123} = (V_{ZD2max} - V_{ZD2min})/R_2$

Hence

$R_1 = (V_{ZD1} - V_{ZD2max})/I_{123}$

$R_3 = V_{ZD2min}/I_{123}$

Below is a Matlab implementation of the calculations showing a demonstration of how this heater works.

```
VZD2=@(T) (T+273.2)/100;
Tamb=70;fr=0.40;V1=12;VOH=V1;VOL=0;
R4=2e3;R2=5e3;R5=6.2e3;R6=1e3;R7=100;
Tmin=50;Tmax=100;aV2=3.5;VZD1=6.8;
VBEQ=0.7;VCEQ=0.17;
VZD2min=VZD2(Tmin);
VZD2max=VZD2(Tmax);
I123=(VZD2max-VZD2min)/R2;
R1=(VZD1-VZD2max)/I123;
R3=VZD2min/I123;
IR5=(VZD1-VZD2(Tamb))/R5;
IR4=(V1-VZD1)/R4;
IZD1=IR4-(I123+IR5);
vp=(fr*R2+R3)/(R1+R2+R3)*VZD1
vn=VZD2(Tamb)
if vp>vn
   vo=VBEQ;
   IBQ=(V1-VBEQ)/R6;
   ICQ=(V1-VCEQ)/R7;
   disp('Heater ON')
else
   VCEQ=V1;
   ICQ=0;
```

```
    disp('Heater OFF')
end
```

```
vp =

    3.4320


vn =

    3.4320


Heater ON
```

The assumed ambient temperature was 70˚C, and the extreme temperatures under consideration are 50˚C and 100˚C. When the temperature is 70˚C or lower, the heater is turned ON. When the temperature is higher than 70˚C, the heater is turned OFF. We also note that when the heater is turned ON, the transistor operates in the saturation region, giving us the highest current the transistor can deliver under these conditions and choices, and $V_{CEQ} = V_{CEsat}$ (here 0.17V).

3.3Window Detector

3.3.1 General Case

If v_{p1} and v_{n1} are associated with U1, and v_{p2} and v_{n2} are associated with U2, then $v_{p1} = V_{TH}$ (in this case 8V), and $v_{n2} = V_{TL}$ (in this case 4V).

$$v_{p1} = V_{TH} = 8V \underset{\substack{< \\ Q_{01}\ \text{Sat}}}{\overset{\substack{Q_{01}\ \text{OFF} \\ >}}{\rule{2em}{0.4pt}}} v_{n1} = v_{in}$$

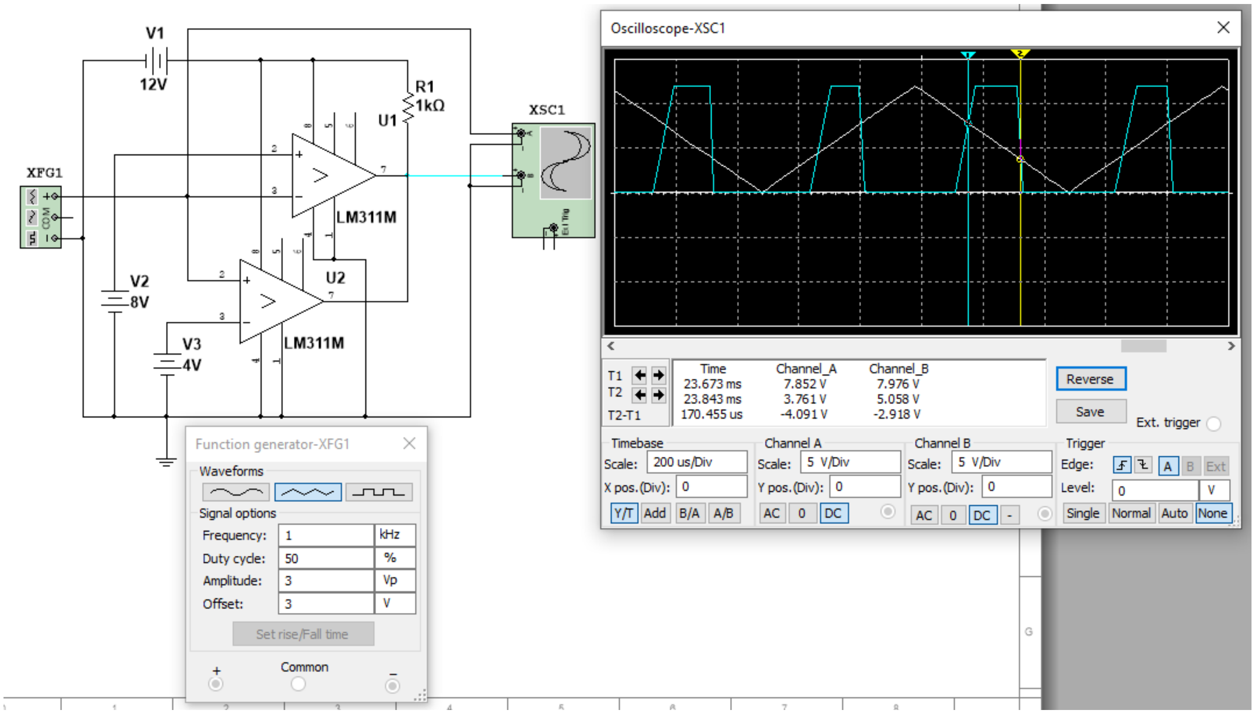
$$v_{p2} = v_{in} \underset{\substack{< \\ Q_{02}\ \text{Sat}}}{\overset{\substack{Q_{02}\ \text{OFF} \\ >}}{\rule{2em}{0.4pt}}} v_{n2} = V_{TL} = 4V$$

When both Q_{01} and Q_{02} are OFF, $v_o = V_{OH}$

When either Q_{01} and Q_{02} or both are saturated, $v_o = V_{OL}$

In this case, $v_o = V_{OH}$ when $V_{TL} < v_{in} < V_{TH}$

and $v_o = V_{OL}$ otherwise, as illustrated in the figure below



	Time	Channel_A	Channel_B
T1	23.673 ms	7.852 V	7.976 V
T2	23.843 ms	3.761 V	5.058 V
T2-T1	170.455 us	-4.091 V	-2.918 V

Timebase Scale: 200 us/Div
X pos.(Div): 0
Channel A Scale: 5 V/Div
Y pos.(Div): 0
Channel B Scale: 5 V/Div
Y pos.(Div): 0
Trigger Level: 0 V

Frequency: 1 kHz
Duty cycle: 50 %
Amplitude: 3 Vp
Offset: 3 V

3.3.2 Power Supply Monitor

1N5222B is a Zener diode with V_Z = 2.5V, and P_{max} = 500mW.

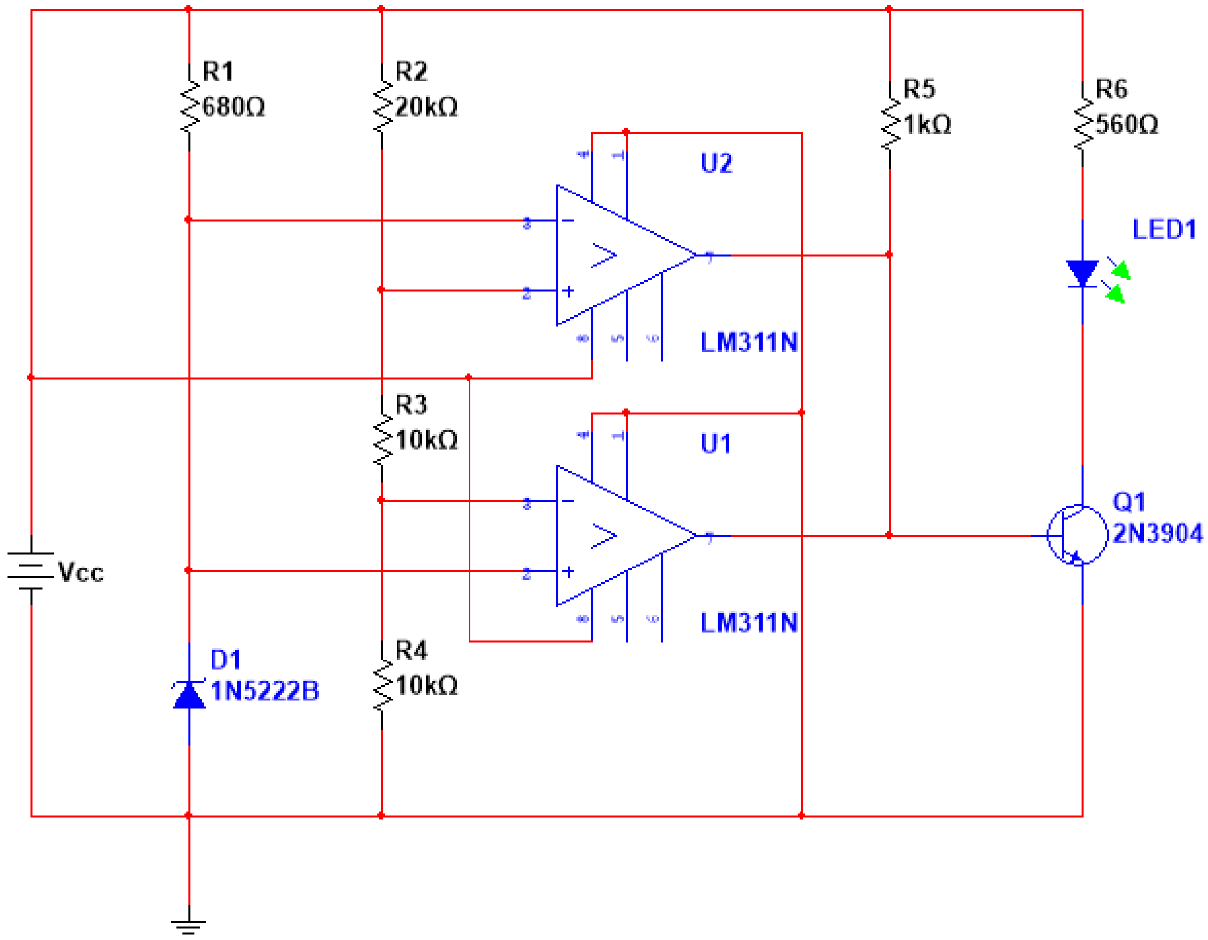
$$v_{p1} = v_{n2} = V_{ZD1}$$

$$v_{n1} = \frac{R_4}{R_2 + R_3 + R_4} V_{CC}$$

$$v_{p2} = \frac{R_3 + R_4}{R_2 + R_3 + R_4} V_{CC}$$

$$v_{p1} = V_{ZD1} = 2.5V \;\underset{Q_{01}\text{ Sat}}{\overset{Q_{01}\text{ OFF}}{\gtrless}}\; v_{n1} = \frac{R_4}{R_2 + R_3 + R_4} V_{CC}$$

$$v_{p2} = \frac{R_3 + R_4}{R_2 + R_3 + R_4} V_{CC} \;\underset{Q_{02}\text{ Sat}}{\overset{Q_{02}\text{ OFF}}{\gtrless}}\; v_{n2} = V_{ZD1} = 2.5V$$



The LED will be ON when both comparators are OFF (Q_1 is saturated)

This will be true when

$$V_{ZD1} > \frac{R_4}{R_2 + R_3 + R_4} V_{CC}$$

$$\frac{R_3 + R_4}{R_2 + R_3 + R_4} V_{CC} > V_{ZD1}$$

This leads to

$$V_{CC} < \frac{R_2 + R_3 + R_4}{R_4} V_{ZD1}$$

$$V_{CC} > \frac{R_2 + R_3 + R_4}{R_3 + R_4} V_{ZD1}$$

In our case, The LED will be ON when 5V < V_{CC} < 10V.

Multisim resulted in the LED being ON in the range 4.9V to 9.9V approximately, and Matlab resulted in the LED being ON in the range 5.1V to 9.9V. A better resolution can be achieved if one so desires.

```
VZD1=2.5;VCC=5.1;VOH=VCC;VOL=0;
R1=680;R2=20e3;R3=10e3;
R4=10e3;R5=1e3;R6=560;
VBEQ=0.7;VCEQsat=0.05;VLED=2.1;
vp1=VZD1;vn2=VZD1;
vn1=VCC*R4/(R2+R3+R4)
vp2=VCC*(R3+R4)/(R2+R3+R4)
I1=VZD1/R1;
I234=VCC/(R2+R3+R4);
IR5=(VCC-VBEQ)/R5;
IR6=(VCC-VLED-VCEQsat)/R6;
if vp1>vn1 && vp2>vn2
   disp('LED ON')
else
   disp('LED OFF')
end
```

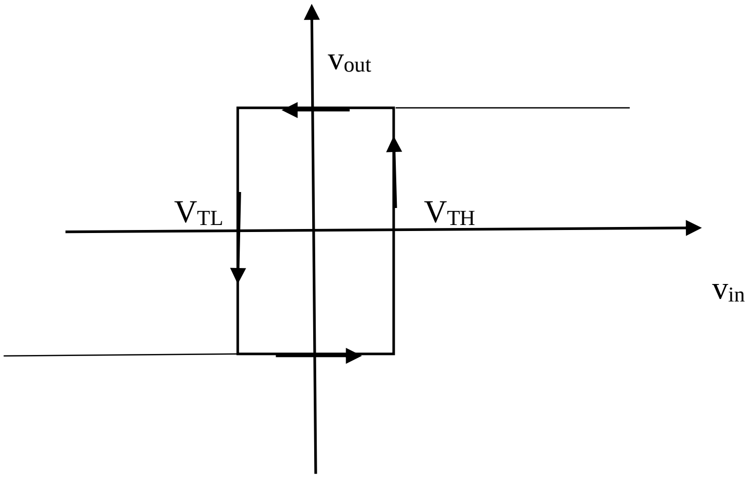
vn1 = 1.2750V

vp2 = 2.5500V

LED ON

The LED is OFF when V_{CC} is outside the desired range (either discharged if V_{CC} is low, or overcharged if V_{CC} is high). Do you think it was a good idea to have the LED OFF outside the desired range? (think of the case when the battery is 8V and yet the LED is OFF)

4. Schmitt Triggers

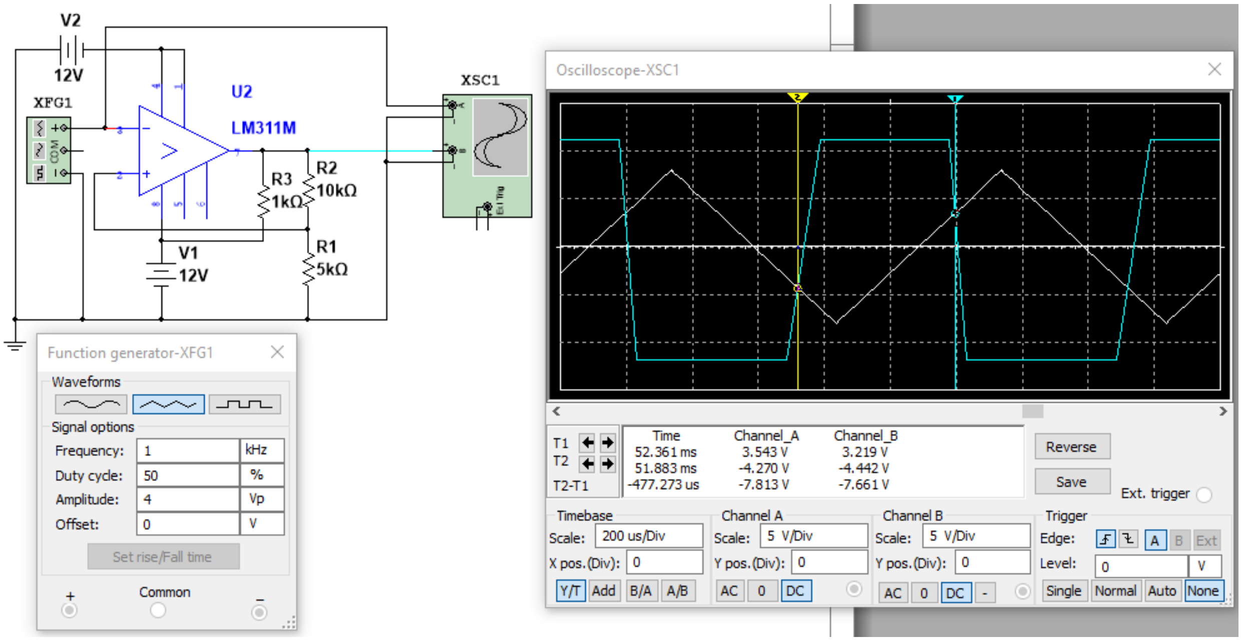


The figure shown above is representative of a Schmitt trigger. The voltage transfer characteristic (v_{out} versus v_{in}), also labeled as a VTC, shows a hysteresis. In this case, when the input is very low (v_{in} much smaller than 0), the output is low. When the input increases, the output remains low until the input exceeds V_{TH}. Then the output switches to a high after that threshold. However, as the input starts decreasing from a very high value, the output remains high until the input becomes smaller than the threshold V_{TL} (not equal to V_{TH}). Then the output switches back to a low. This transfer characteristic is also representative of a non-inverting Schmitt trigger because the output is low when the input is low, and the output is high when the input is high.

This VTC will be achieved using a comparator and positive feedback.

4.1 Simple Inverting Schmitt Trigger

The figure below shows a simple <u>inverting</u> Schmitt trigger (the output is high when the input is low and the output is low when the input is high).



Frequency: 1 kHz
Duty cycle: 50 %
Amplitude: 4 Vp
Offset: 0 V

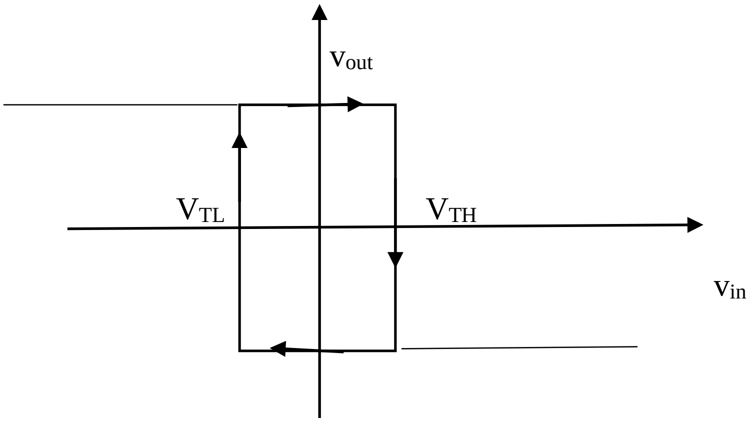
```
        Time          Channel_A    Channel_B
T1      52.361 ms     3.543 V      3.219 V
T2      51.883 ms     -4.270 V     -4.442 V
T2-T1   -477.273 us   -7.813 V     -7.661 V
```

Timebase
Scale: 200 us/Div
X pos.(Div): 0

Channel A
Scale: 5 V/Div
Y pos.(Div): 0

Channel B
Scale: 5 V/Div
Y pos.(Div): 0

Trigger
Edge: A
Level: 0 V
Ext. trigger



$v_n = v_{in}$

$v_p = R_1/(R_1+R_2)*v_o$

We are able to use voltage division because even though the two resistors share their node with the non-inverting input, the current through the non-inverting input being 0 because we assumed the comparator to have an infinite input resistance allows us to think as the two resistors to be virtually in series (same current).

When v_{in} is very small (negative range), the VTC shows that the output is high. If we had assumed the output to be low, we would have found that state to be unstable, and the output will switch to a stable state, which in this case is high.

$$v_n = v_{in} \underset{Q_0\ \text{OFF}}{\overset{Q_0\ \text{Sat}}{\gtrless}} v_p = \frac{R_1}{R_1 + R_2} v_o$$

Note that when Q_0 is OFF, $v_o = V_{OH} = V_{CC}$, though in practical circuitry, the output may be lower than V_{CC} due to internal voltage drops.

When Q_0 is saturated, $v_o = V_{OL} = -V_{EE}$

Hence, the switching occurs when v_{in} exceeds the threshold

$$V_{TH} = \frac{R_1}{R_1 + R_2} V_{OH} = \frac{R_1}{R_1 + R_2} V_{CC}$$

Conversely, as the input is very high, and the stable state of the output is low where $v_o=V_{OL}=-V_{EE}$, the switch will occur when the input becomes lower than the threshold

$$V_{TL} = \frac{R_1}{R_1 + R_2} V_{OL} = -\frac{R_1}{R_1 + R_2} V_{EE}$$

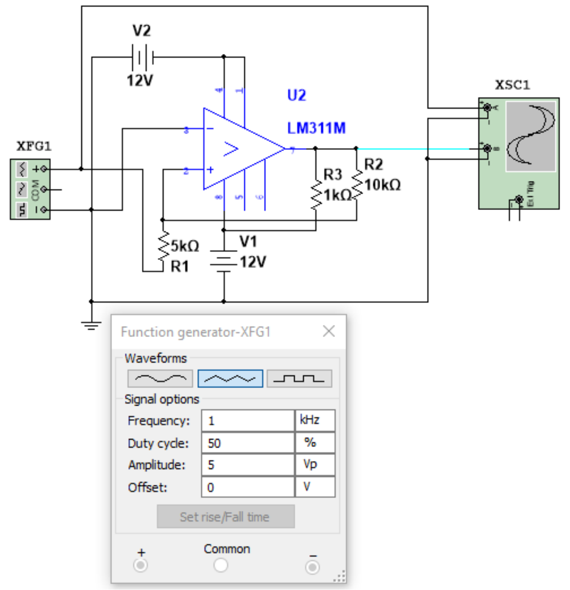
Note that the thresholds are equal in magnitude and opposite in sign if we assume dual power supply and $V_{CC} = V_{EE}$.

4.2Simple Noninverting Schmitt Trigger

The figure below shows a simple noninverting Schmitt trigger (the output is low when the input is low and the output is high when the input is high).

$v_n = 0$

$v_p = R_1/(R_1+R_2)*v_o + R_2/(R_1+R_2)*v_{in}$

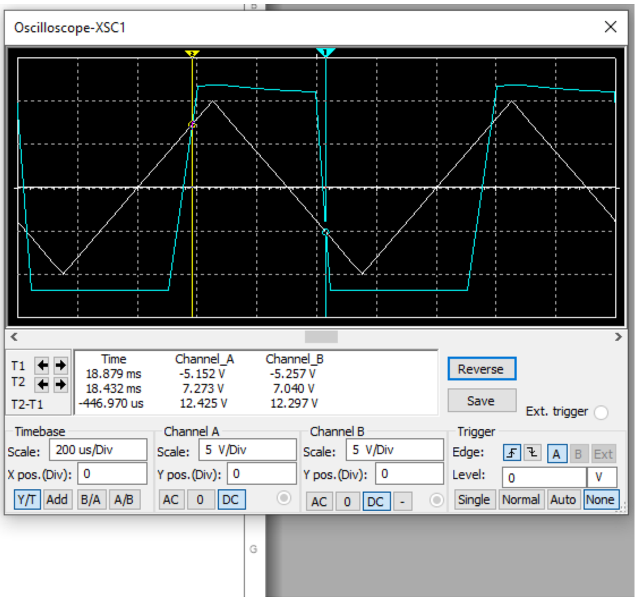


Function generator-XFG1

Waveforms

Signal options

Frequency:	1	kHz
Duty cycle:	50	%
Amplitude:	5	Vp
Offset:	0	V



Oscilloscope-XSC1

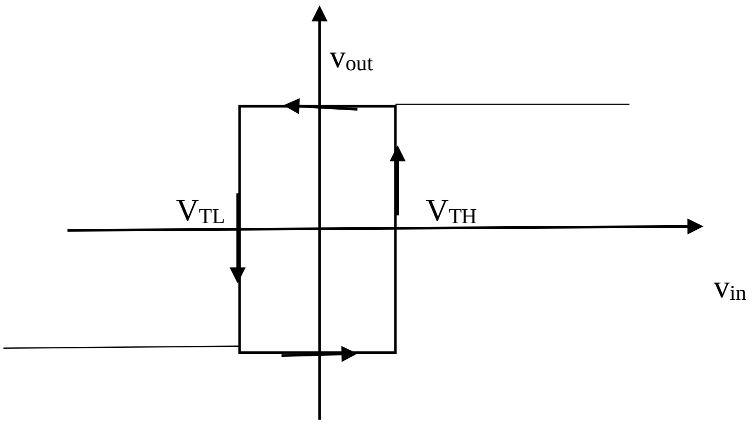
	Time	Channel_A	Channel_B
T1	18.879 ms	-5.152 V	-5.257 V
T2	18.432 ms	7.273 V	7.040 V
T2-T1	-446.970 us	12.425 V	12.297 V

Timebase: Scale: 200 us/Div; X pos.(Div): 0

Channel A: Scale: 5 V/Div; Y pos.(Div): 0

Channel B: Scale: 5 V/Div; Y pos.(Div): 0

Trigger: Edge: A; Level: 0 V



$$v_n = 0 \mathrel{\substack{Q_0\ \text{Sat} \\ > \\ < \\ Q_0\ \text{OFF}}} v_p = \frac{R_1}{R_1+R_2} v_o + \frac{R_2}{R_1+R_2} v_{in}$$

We assume that we start with the appropriate assumption that the output is low when the input is low.

Hence the output will switch to a high when the input reaches V_{TH}

$$\frac{R_1}{R_1+R_2}V_{OL}+\frac{R_2}{R_1+R_2}V_{TH}=0$$

$$\text{or}\, V_{TH}=-\frac{R_1}{R_2}V_{OL}$$

When the input is very high, the output is high. As the input decreases toward and reaches V_{TL},

$$\frac{R_1}{R_1+R_2}V_{OH}+\frac{R_2}{R_1+R_2}V_{TL}=0$$

$$\text{or}\, V_{TL}=-\frac{R_1}{R_2}V_{OH}$$

with the width of the hysteresis becoming equal to

$$\Delta V_T=\frac{R_1}{R_2}(V_{OH}-V_{OL})$$

We again see that the thresholds are equal in magnitude and opposite in sign if we assume dual power supply and $V_{CC} = V_{EE}$.

4.3Single Power Supply Inverting Schmitt Trigger

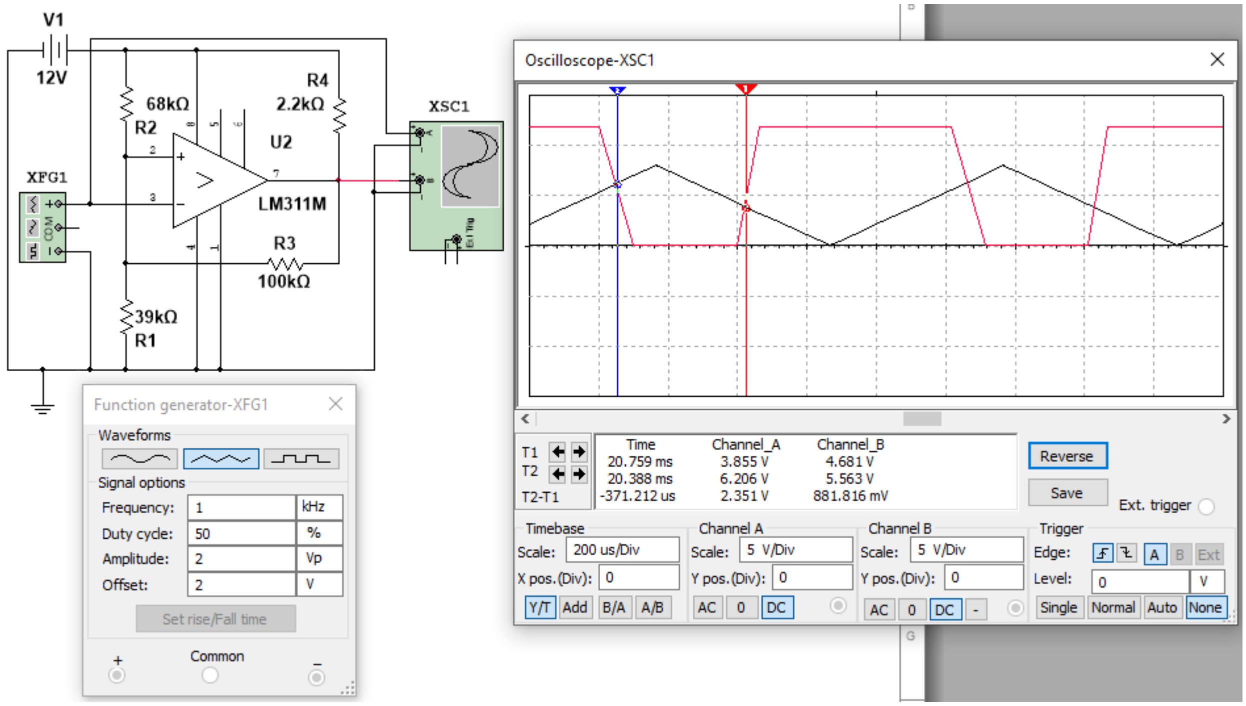
We again assume that the output is high when the input is low and the output is low when the input is high (stable state)

$$v_n=v_{in}$$

Using superposition, with V_{CC} and v_o being the two sources affecting v_p

$$v_p=\frac{R_1//R_3}{R_1//R_3+R_2}V_{CC}+\frac{R_1//R_2}{R_1//R_2+R_3}v_o$$

$$v_n=v_{in}\ \underset{\text{Q}_0\text{ OFF}}{\overset{\text{Q}_0\text{ Sat}}{\frac{>}{<}}}\ v_p=\frac{R_1//R_3}{R_1//R_3+R_2}V_{CC}+\frac{R_1//R_2}{R_1//R_2+R_3}v_o$$



Function generator-XFG1

Waveforms: triangle

Signal options

Frequency: 1 kHz

Duty cycle: 50 %

Amplitude: 2 Vp

Offset: 2 V

Oscilloscope-XSC1

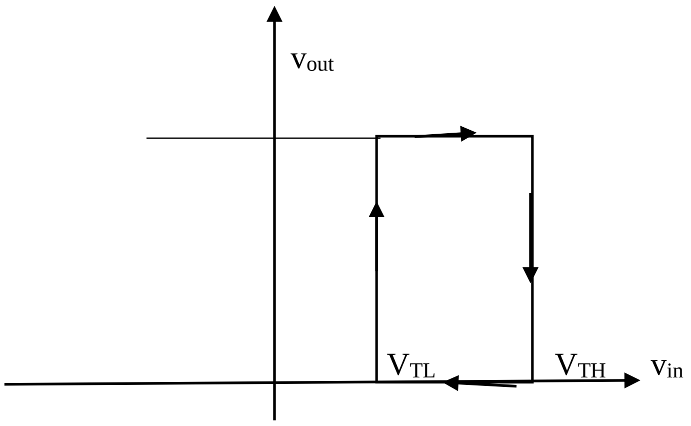
	Time	Channel_A	Channel_B
T1	20.759 ms	3.855 V	4.681 V
T2	20.388 ms	6.206 V	5.563 V
T2-T1	-371.212 us	2.351 V	881.816 mV

Timebase: Scale: 200 us/Div; X pos.(Div): 0; Y/T

Channel A: Scale: 5 V/Div; Y pos.(Div): 0; DC

Channel B: Scale: 5 V/Div; Y pos.(Div): 0; DC

Trigger: Edge: rising, A; Level: 0 V; None



Hence the output will switch to a low when the input reaches V_{TH}

$$\frac{R_1//R_3}{R_1//R_3+R_2}V_{CC}+\frac{R_1//R_2}{R_1//R_2+R_3}V_{OH}=V_{TH}$$

Conversely, the output is low when the input is high.

Hence the output will switch to a high when the input reaches V_{TL}

$$\frac{R_1//R_3}{R_1//R_3+R_2}V_{CC}+\frac{R_1//R_2}{R_1//R_2+R_3}V_{OL}=V_{TL}$$

Knowing that $\mathrm{V_{OH}=V_{CC},V_{OL}=0}$

$$V_{TH}=\frac{R_1//R_3}{R_1//R_3+R_2}V_{CC}+\frac{R_1//R_2}{R_1//R_2+R_3}V_{CC}=\left(\frac{R_1//R_3}{R_1//R_3+R_2}+\frac{R_1//R_2}{R_1//R_2+R_3}\right)V_{CC}$$

$$V_{TL}=\frac{R_1//R_3}{R_1//R_3+R_2}V_{CC}$$

$$\begin{aligned}\frac{R_1//R_3}{R_1//R_3+R_2}+\frac{R_1//R_2}{R_1//R_2+R_3}&=\frac{\frac{R_1R_3}{R_1+R_3}}{\frac{R_1R_3}{R_1+R_3}+R_2}+\frac{\frac{R_1R_2}{R_1+R_2}}{\frac{R_1R_2}{R_1+R_2}+R_3}\\&=\frac{R_1R_3}{R_1R_3+R_2(R_1+R_3)}+\frac{R_1R_2}{R_1R_2+R_3(R_1+R_2)}\\&=\frac{R_1R_3+R_1R_2}{R_1R_3+R_2(R_1+R_3)}=\frac{R_1(R_3+R_2)}{R_1(R_2+R_3)+R_2R_3}\\&=\frac{R_1}{R_1+\frac{R_2R_3}{(R_2+R_3)}}=\frac{R_1}{R_1+R_2//R_3}\end{aligned}$$

$$V_{TH}=\frac{R_1}{R_1+R_2//R_3}V_{CC}$$

$$V_{TL}=\frac{R_1//R_3}{R_1//R_3+R_2}V_{CC}$$

The design of this Schmitt trigger, with the choice of the resistor values to satisfy a specific set of $\mathrm{V_{TL}}$ and $\mathrm{V_{TH}}$, requires the selection of 4 resistor values with only 2 equations. Hence, we will choose a value for $\mathrm{R_3}$ and the value of $\mathrm{R_4}$ will be chosen to make sure v_o is very close to $\mathrm{V_{CC}}$ ($R_3 \gg R_4$).

The remaining resistor values will be obtained with the help of the following equations

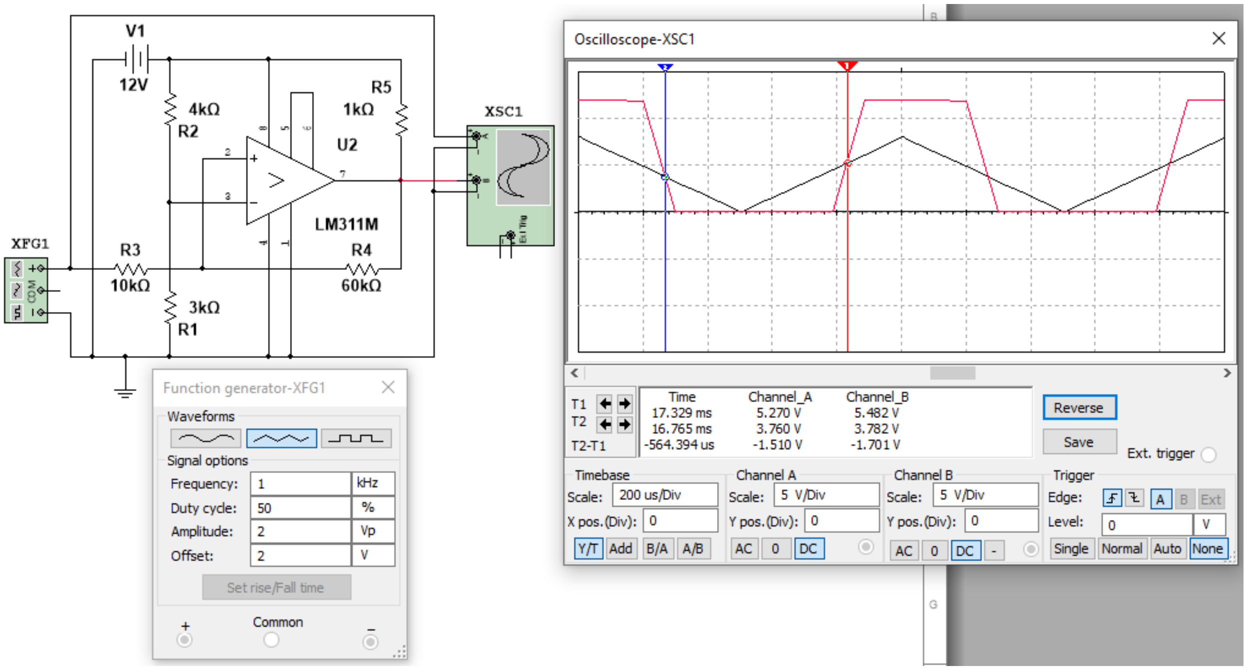
$$\frac{1}{R_1} = \frac{V_{CC} - V_{TH}}{V_{TH}} \left(\frac{1}{R_2} + \frac{1}{R_3} \right)$$

$$\frac{1}{R_2} = \frac{V_{TL}}{V_{CC} - V_{TL}} \left(\frac{1}{R_1} + \frac{1}{R_3} \right)$$

Example:

Assume that we are using a single power supply V_{CC} = 12V, and made a choice of V_{TL} = 3.5V and V_{TH} = 6V. If we start with the choice of R_4 = 2.2kΩ, then R_3 = 100 kΩ would be suitable. In that case, Approximate standard values of R_1 and R_2 satisfying the previous equations would be R_1 = 39 kΩ, and R_2 = 68 kΩ, which would complete the design (achieving V_{TL} = 3.5V and V_{TH} = 5.89V).

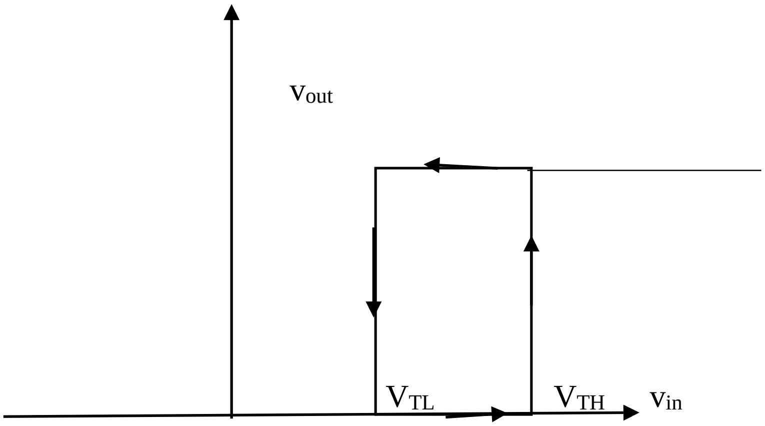
4.4Single Power Supply Noninverting Schmitt Trigger



	Time	Channel_A	Channel_B
T1	17.329 ms	5.270 V	5.482 V
T2	16.765 ms	3.760 V	3.782 V
T2-T1	-564.394 us	-1.510 V	-1.701 V

Timebase: Scale: 200 us/Div; X pos.(Div): 0; Y/T
Channel A: Scale: 5 V/Div; Y pos.(Div): 0; DC
Channel B: Scale: 5 V/Div; Y pos.(Div): 0; DC
Trigger: Edge: A; Level: 0 V; None

Frequency: 1 kHz
Duty cycle: 50 %
Amplitude: 2 Vp
Offset: 2 V



$$v_n = \frac{R_1}{R_1+R_2}V_{CC}$$

Using superposition, with V_{CC} and v_o being the two sources affecting v_p

$$v_p = \frac{R_4}{R_3+R_4}v_{in} + \frac{R_3}{R_3+R_4}v_o$$

$$v_n = \frac{R_1}{R_1+R_2}V_{CC} \underset{Q_0\ \text{OFF}}{\overset{Q_0\ \text{Sat}}{\gtrless}} v_p = \frac{R_4}{R_3+R_4}v_{in} + \frac{R_3}{R_3+R_4}v_o$$

With the assumption that when the input iss low, the output is low (stable state), and $V_{OH} = V_{CC}$, and $V_{OL} = 0$, as v_{in} increases from a low, the output will switch when the input reaches V_{TH}. Conversely, the output will switch with decreasing input when the input reaches V_{TL}.

When the input reaches V_{TH} while increasing,

$$v_n = \frac{R_1}{R_1+R_2}V_{CC} = v_p = \frac{R_4}{R_3+R_4}V_{TH}$$

Conversely, when the input reaches V_{TL} while decreasing,

$$v_n = \frac{R_1}{R_1+R_2}V_{CC} = v_p = \frac{R_4}{R_3+R_4}V_{TL} + \frac{R_3}{R_3+R_4}V_{CC}$$

giving us

$$V_{TH} = \frac{R_3 + R_4}{R_1 + R_2} \frac{R_1}{R_4} V_{CC}$$

$$V_{TL} = \frac{R_1}{R_4} \left(\frac{R_3 + R_4}{R_1 + R_2} - \frac{R_3}{R_1} \right) V_{CC}$$

One can also show that

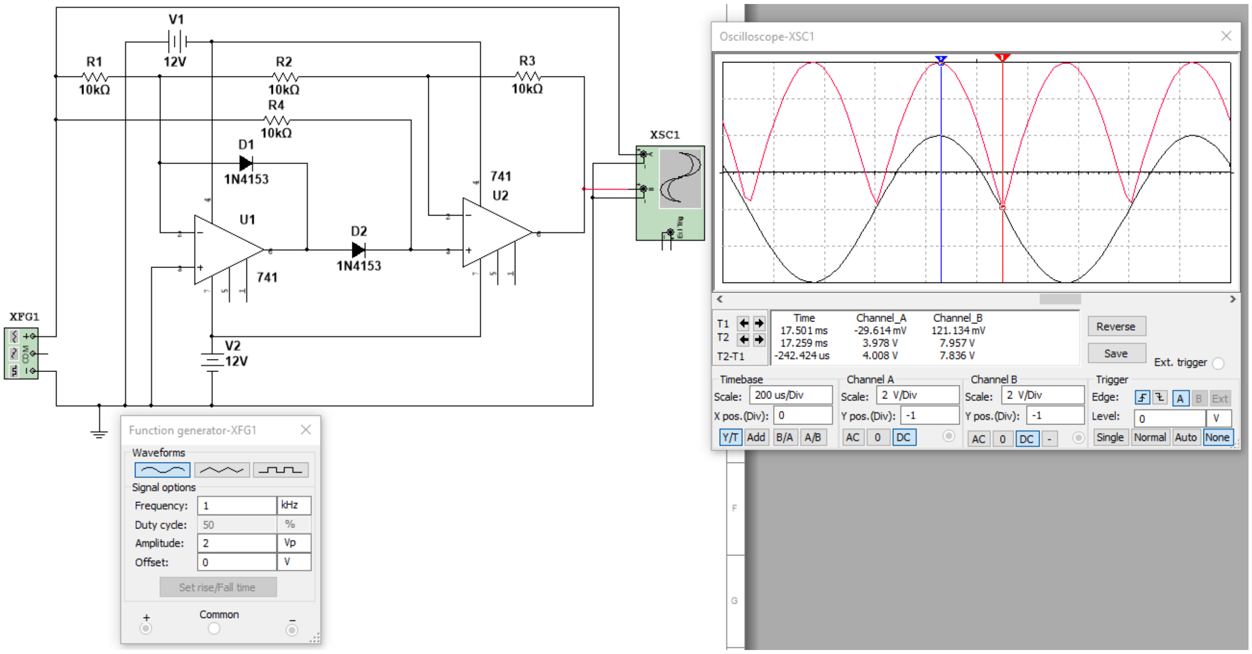
$$\frac{R_3}{R_4} = \frac{V_{TH} - V_{TL}}{V_{CC}}$$

$$\frac{R_2}{R_1} = \frac{V_{CC} - V_{TL}}{V_{TH}}$$

We can see that with the components values in the given circuit

V_{TL} = 4V, and V_{TH} = 6V

5. Full Wave Rectifier



	Time	Channel_A	Channel_B
T1	17.501 ms	-29.614 mV	121.134 mV
T2	17.259 ms	3.978 V	7.957 V
T2-T1	-242.424 us	4.008 V	7.836 V

Ext. trigger ○

Timebase
Scale: 200 us/Div
X pos. (Div): 0

Channel A
Scale: 2 V/Div
Y pos. (Div): -1

Channel B
Scale: 2 V/Div
Y pos. (Div): -1

Trigger
Edge: A
Level: 0 V

Signal options
Frequency: 1 kHz
Duty cycle: 50 %
Amplitude: 2 Vp
Offset: 0 V

Let us assume that $R_1 = R_2 = R$, and that $R_3 = (A-1)R$

When $v_{in} > 0$, D_1 is ON. Since we have a negative feedback connection, this means that $v_{n1}=v_{p1}=0V$. Hence $v_{01} = -V_{D1on}$. In that case, D_2 is OFF since the anode voltage iss definitely less than the cathode voltage (cathode connected to v_{in} through R_4). Since v_{in} is applied to v_{p2} through R_4, and $v_{n1}= 0V$, the second stage acts like a noninverting amplifier with a gain equal to v_o/v_{in}.

$$With\ R_1=R_2=R\ and\ R_3=(A-1)R$$

$$\frac{v_o}{v_{in}}=1+\frac{R_3}{R_2}=1+\frac{(A-1)R}{R}=A \quad \text{when}\, v_{in}>0$$

When $v_{in} < 0$, D_1 is OFF. The biasing of D_2 by a negative v_{in} through R_4 turns it ON since the inverting input of the first OPAMP is still ground because of negative feedback (from D_2 to 2nd OPAMP to R_3 to R_2. Hence

$$\frac{0-v_{in}}{R_1}=\frac{v_o-0}{R_2+R_3}$$

$$\text{Hence} \quad \frac{v_o}{v_{in}}=-\frac{R_2+R_3}{R_1} \quad \text{when}\, v_{in}<0$$

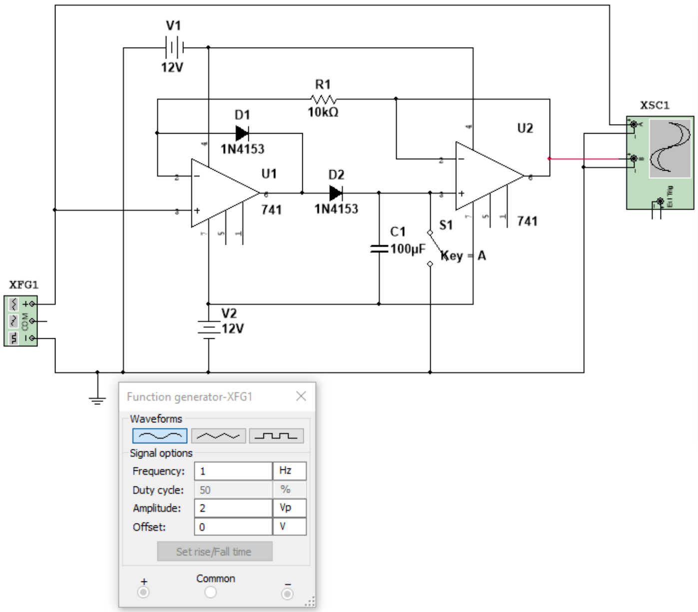
$$\text{With } R_1=R_2=R \text{ and } R_3=(A-1)R$$

$$\frac{v_o}{v_{in}}=-A \quad \text{when}\, v_{in}<0$$

$$\text{Hence } v_o=A|v_{in}| \quad \text{and a full wave rectification ensues}$$

6. Peak Detector

When the switch is closed, the output follows the input waveform. When the switch is closed, the output tracks and remains equal to the highest peak it had encountered.



Function generator-XFG1

Waveforms

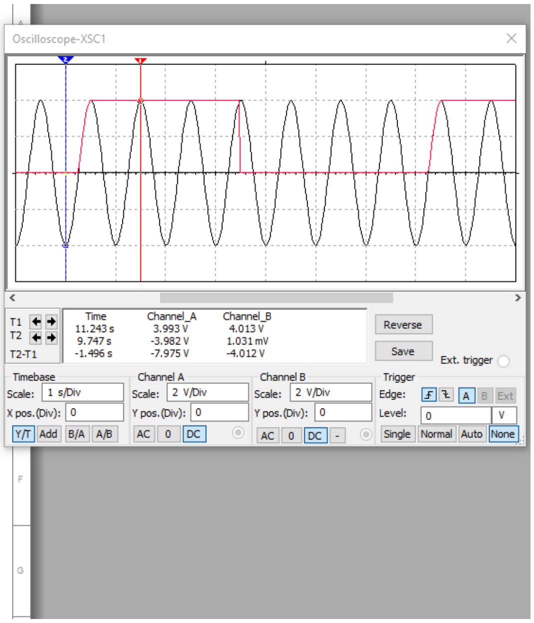
Signal options

Frequency: 1 Hz

Duty cycle: 50 %

Amplitude: 2 Vp

Offset: 0 V



Oscilloscope-XSC1

	Time	Channel_A	Channel_B
T1	11.243 s	3.993 V	4.013 V
T2	9.747 s	-3.982 V	1.031 mV
T2-T1	-1.496 s	-7.975 V	-4.012 V

Timebase: Scale: 1 s/Div; X pos.(Div): 0

Channel A: Scale: 2 V/Div; Y pos.(Div): 0

Channel B: Scale: 2 V/Div; Y pos.(Div): 0

Trigger: Edge: A; Level: 0 V