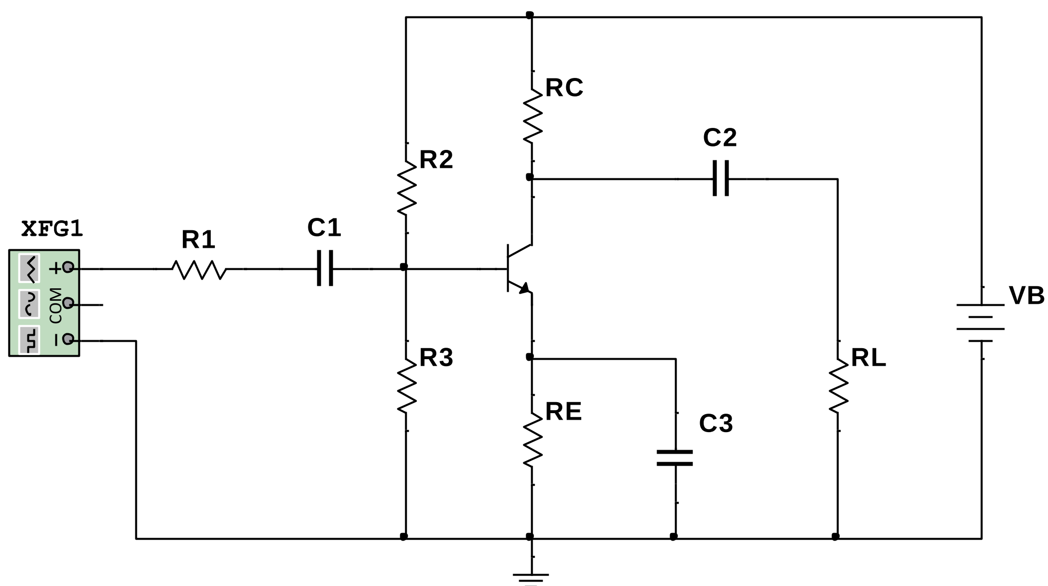
Chapter 2

Common Emitter and Common Source Configurations

1. Coupling and Bypass Capacitors



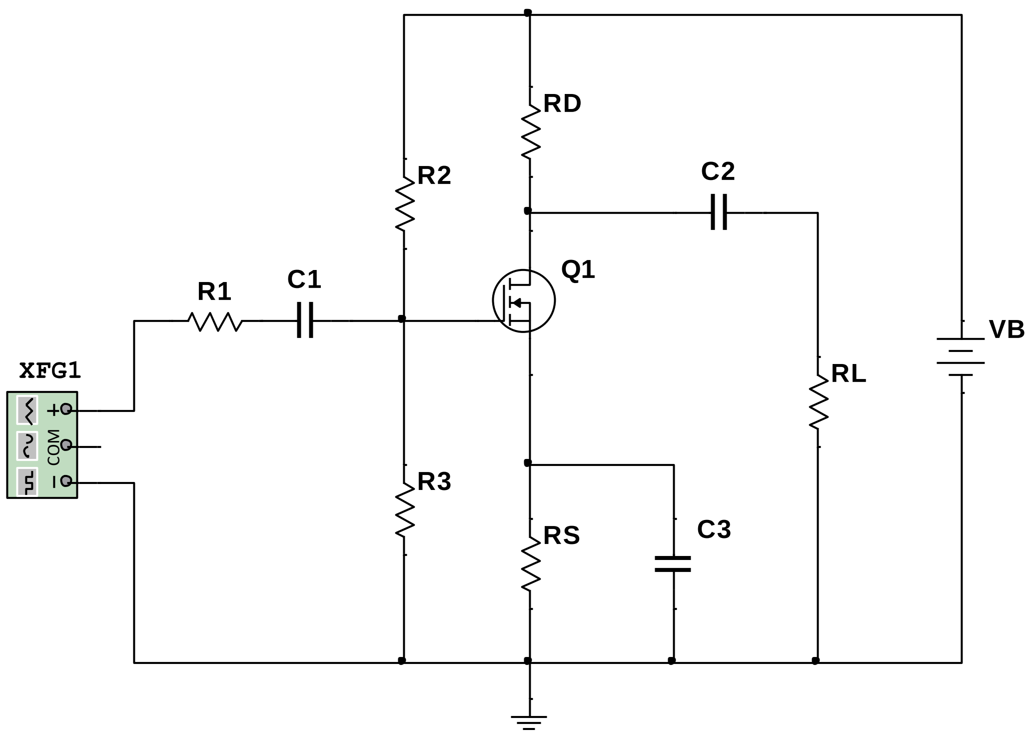
C_1 and C_2 are coupling capacitors, and C_3 is a bypass capacitor. Their impedances are ideally zero for any frequency except DC, when they are open. Theoretically, their capacitances are supposed to be infinite.

The coupling capacitors are supposed to isolate this stage from the previous or subsequent stages from a DC point of view. Its Q-point is supposed to remain the same no matter what this stage is connected to.

The bypass capacitor is supposed to provide a short across R_E from an AC point of view, giving us a higher voltage gain if needed. Since R_E is needed to stabilize the Q-point, we cannot just replace it with a short, unless this stage is part of a circuit that needs to be integrated, in which case we know what stages will be connected to this one. We will see in subsequent topics how we can try to achieve both.

There is a price to be paid when using capacitors. The utilization of electrolytic capacitors, if the capacitances need to be large, is a major problem because they tend to degrade faster than the other components, sometimes affecting an entire board. In addition, there are the economic, and size of the board components. The worst part is that we cannot amplify DC signals, and the signals that can be amplified with an appropriate gain have to have frequencies higher than some frequency f_L, which will be discussed in the frequency analysis of voltage amplifiers.

The same reasons apply to a MOSFET amplifier as shown below:



2. DC Equivalents and Analysis of Amplifiers

A. BJT

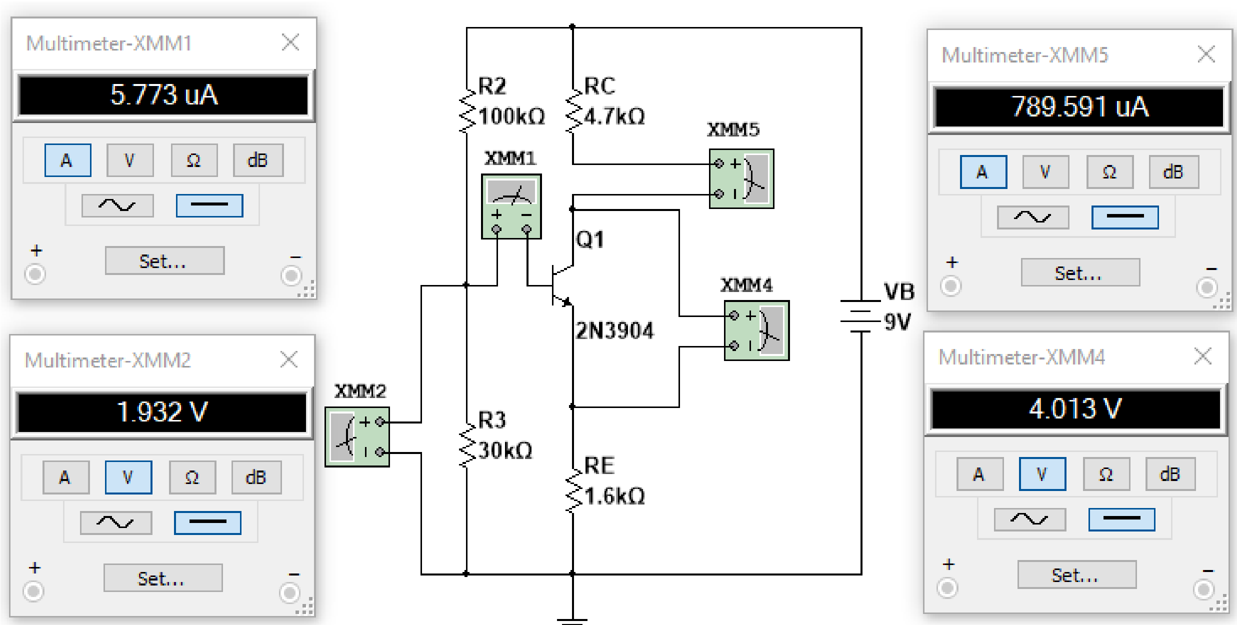
Since we are dealing only with DC power, the capacitors have an infinite impedance and become open circuits. Any component that does not affect the DC analysis is removed from the circuit.

Let us first replace the combination of R_2, R_3, and V_B with their Thevenin equivalent.

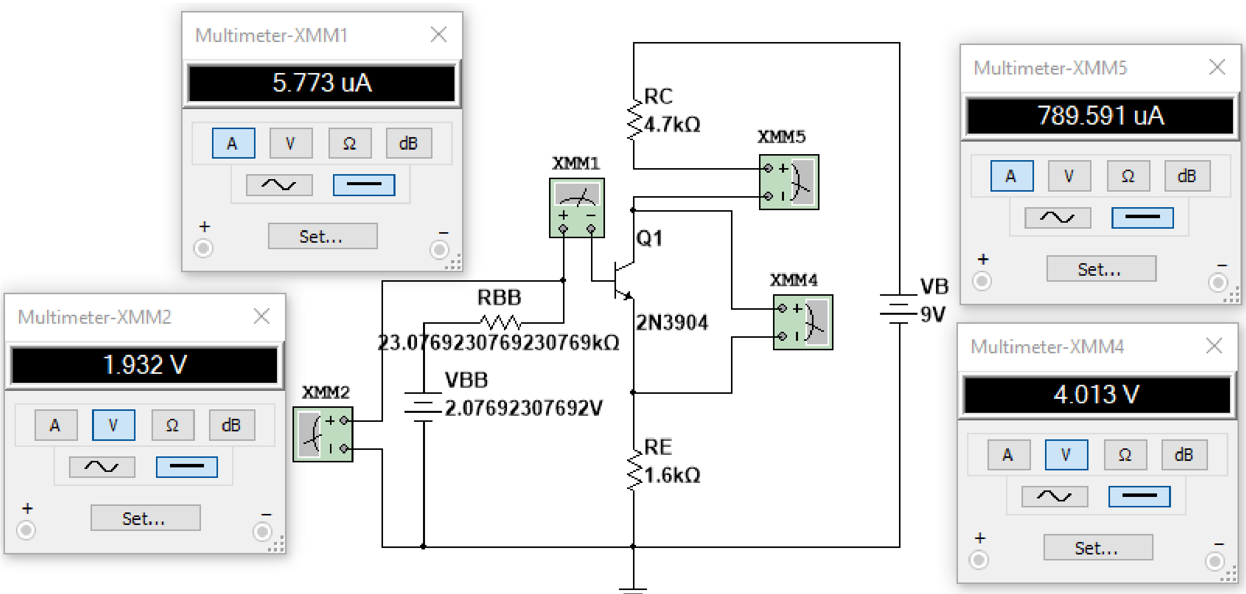
$$V_{BB} = \frac{R_3}{R_2 + R_3} V_B$$

$$R_{BB} = \frac{R_2 R_3}{R_2 + R_3}$$

Giving us, $V_{BB} = 2.08V$
$R_{BB} = 23.08k\Omega$



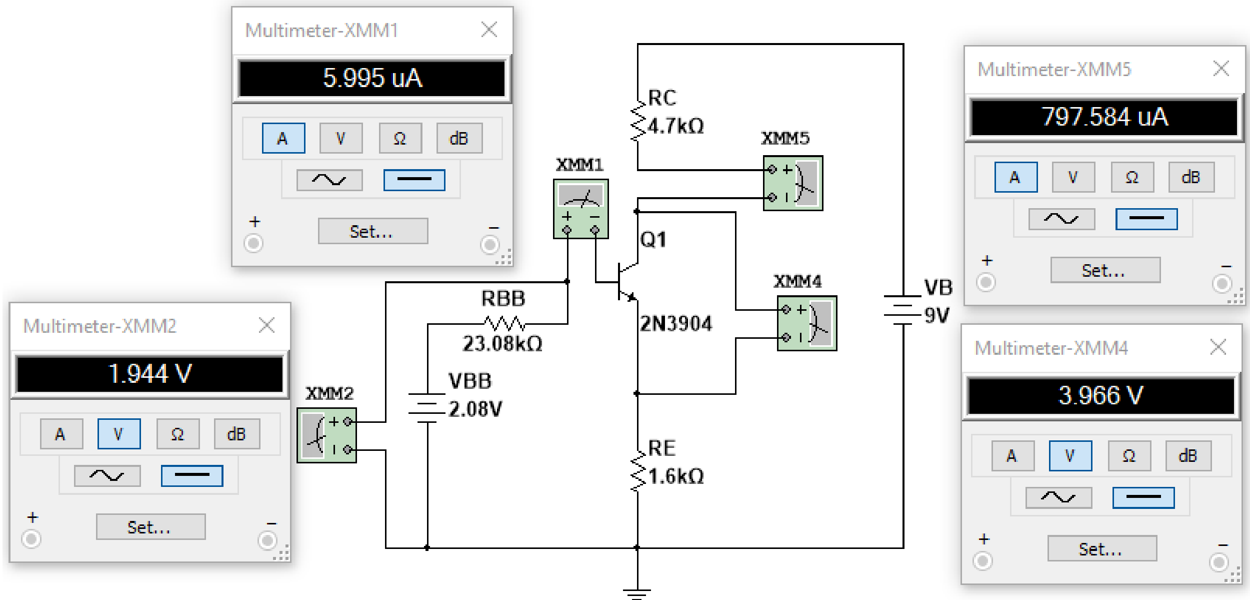
Replacing that combination with very accurate evaluations of V_{BB} and R_{BB}, allowed us to see that the two circuits were completely equivalent of each other. However, using the approximations within two digits after the decimal point, we have results that deviate from the original values by an amount that is acceptable.



We will from now on, accept the minor variations due to the replacements of values within 1% or less from their exact values.

Note that this simulation assumed the transistor had a $\beta_{max} = 416$.

Writing KVL equations for the left loop and the right loop, and for simplicity, especially with this large β, we will assume that $I_E = I_C$, and that $V_{BEQ} = 0.7V$ (this will be assumed during the entire course, except where noted).



$$-V_{BB} + R_{BB}\frac{I_C}{\beta} + V_{BEQ} + R_E I_C = 0$$

$$-V_B + R_C I_C + V_{CEQ} + R_E I_C = 0$$

```
R2=100e3;R3=30e3;RB=1/(1/R2+1/R3);
VBEQ=0.7;RC=4.7e3;RE=1.6e3;
beta=136;VB=9;VBB=R3/(R2+R3)*VB;
A=[RB/beta+RE,0;RC+RE,1];
B=[VBB-VBEQ;VB];
IVcq=A\B;
disp(['The Q-point is ICQ = ',num2str(IVcq(1)*1000),' mA'])
disp(['            VCEQ = ',num2str(IVcq(2)),' V'])
```

```
The Q-point is ICQ = 0.77806 mA
            VCEQ = 4.0982 V
```

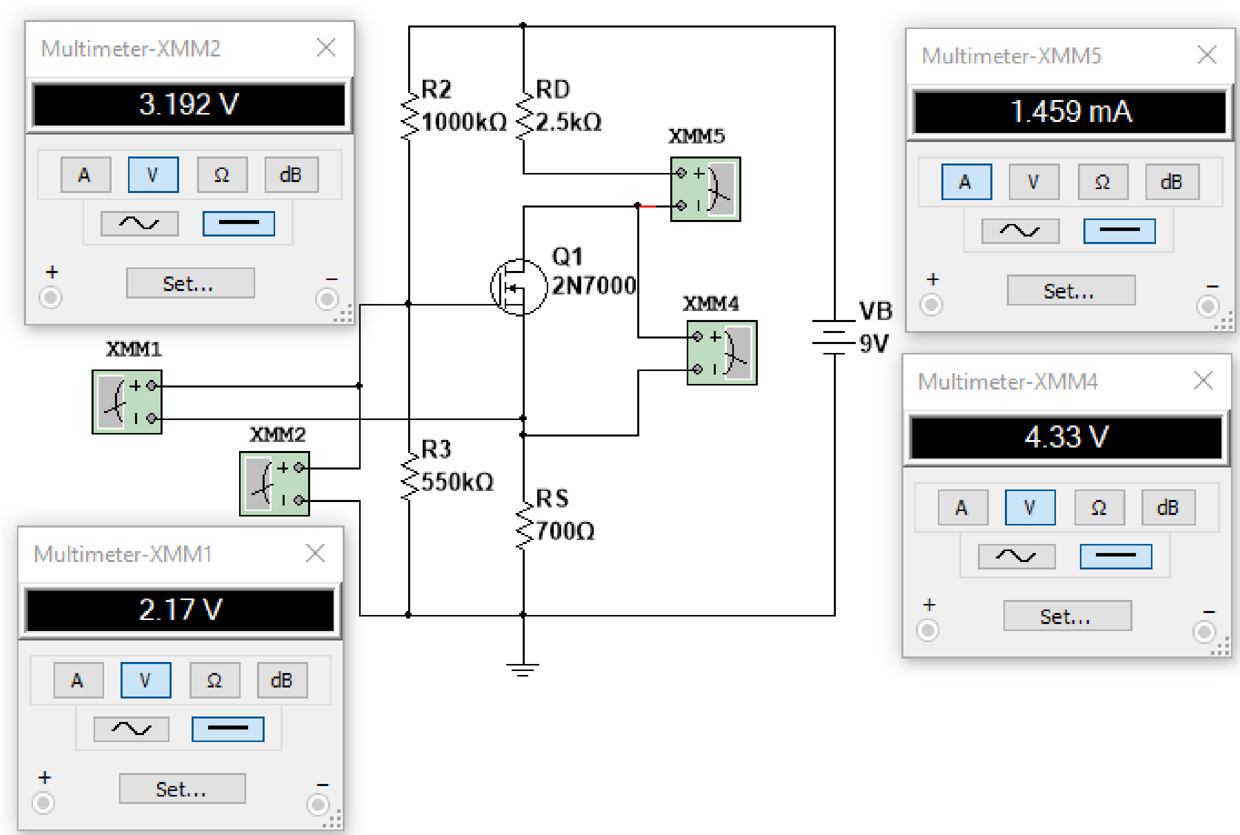
These numbers are close to the simulated values because we adjusted the beta to what the values of I_B and I_C correspond to. Even though the model uses an ideal max of 416, that is definitely not the ratio that the simulation uses since the values will be visibly different from the results of the simulation and the calculations.

```
R2=100e3;R3=30e3;RB=1/(1/R2+1/R3);
VBEQ=0.7;RC=4.7e3;RE=1.6e3;
beta=416;VB=9;VBB=R3/(R2+R3)*VB;
A=[RB/beta+RE,0;RC+RE,1];
B=[VBB-VBEQ;VB];
IVcq=A\B;
disp(['The Q-point is ICQ = ',num2str(IVcq(1)*1000),' mA'])
disp(['              VCEQ = ',num2str(IVcq(2)),' V'])
```

```
The Q-point is ICQ = 0.83174 mA
              VCEQ = 3.76 V
```

Even adjusting V_{BEQ} to 0.668V did not provide a major correction. Hence we see the influence of the current gain on the establishment of the Q-point, and later we will design our amplifiers so that the current gain does not affect negatively our specifications, unless we are in the case where we desire a very high gain regardless of what the value may be. Hence, the goal is to be able to use transistors without having to test their current gains. The presence of R_E makes that possible.
In this case, if R_E were to be a short circuit, the circuit will operate in the saturation region. When we lowered R_C to 1kΩ, the circuit became active again but operating near V_{CEsat}, making it unsuitable for voltage amplification.

B. MOSFET



The 2N7000 is a low current general purpose N-channel enhancement-mode MOSFET with a threshold voltage V_{TN} that varies between 0.8 and 3V. However, the default setting for Multisim is 2V, which can be verified since for $V_{GS} < 2V$, the current I_{DS} becomes zero. It was also noted that when an ammeter was used to measure the gate current (which should have been zero), the results were affected by its presence, meaning that the ammeter is not a true short circuit (or as used by Multisim).
The value of K_n that will be used in the analysis of this circuit was obtained from one simulation of the circuit. We see that for $V_{GS} = 22.17V$, $V_{TN} = 2V$, the current was 1.459mA. Since the capacitors are open in DC, we have the circuit shown above.
Since in the saturation region

$$I_{DS} = \frac{K_n}{2}(V_{GS} - V_{TN})^2$$

This gave us $K_n = 0.1A/V^2$
Let us now perform the DC analysis of the circuit shown above. Assume that the circuit operates in the saturation region ($V_{DS} > V_{GS} - V_{TN} > 0$)

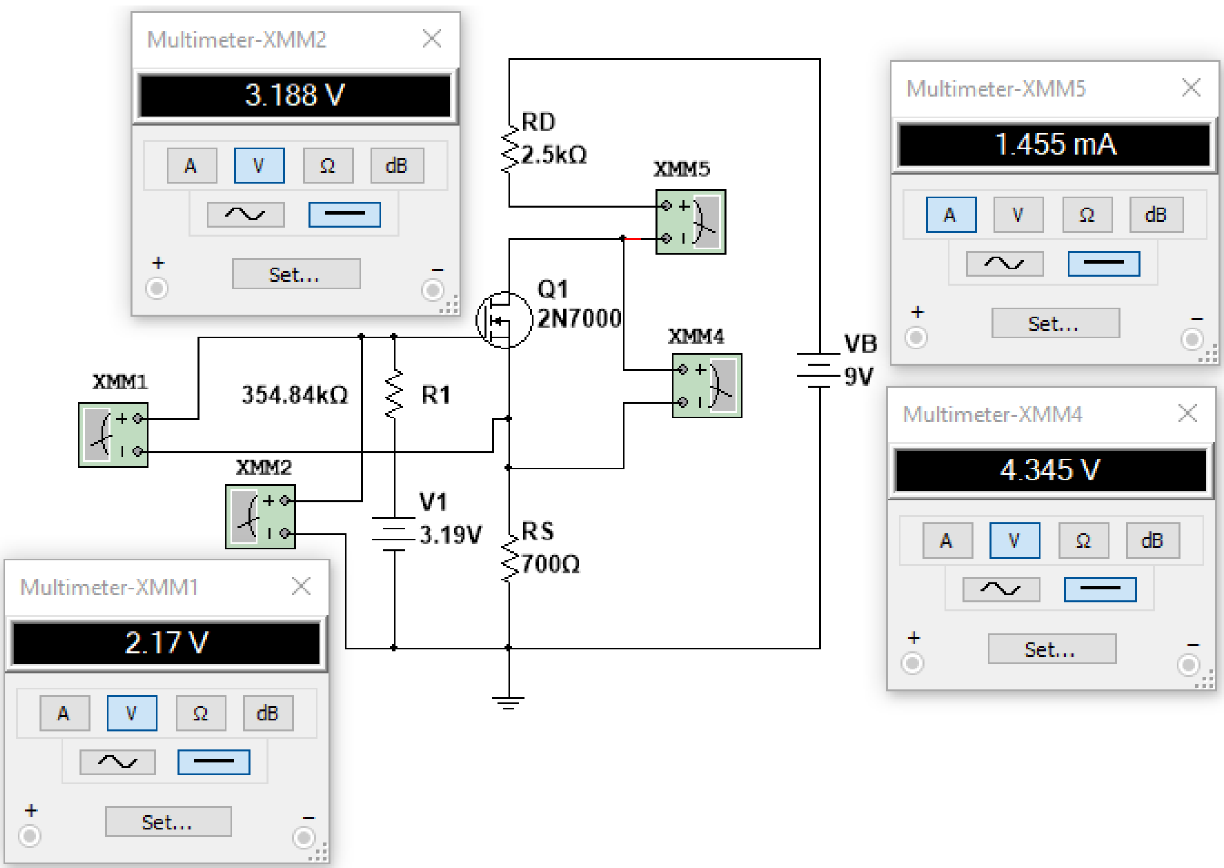
If we replace the combination R_2, R_3, and V_B with their Thevenin equivalent.

$$V_{GG} = \frac{R_3}{R_2 + R_3} V_B$$

$$R_{GG} = \frac{R_2 R_3}{R_2 + R_3}$$

Giving us, $V_{GG} = 3.19V$

$R_{GG} = 354.84k\Omega$



Except for very few minor variations due to the approximations used in evaluating V_{GG}, and R_{GG}, this is an equivalent circuit.

Since $I_G = 0$, $V_{GG} = V_{GS} + R_S I_{DS} = 3.19V$

$$I_{DS} = \frac{K_n}{2}(V_{GS} - V_{TN})^2$$

Substitution of V_{GS} from the first equation into the second equation leads to

$$R_S^2 I_{DS}^2 - (2(V_{GG} - V_{TN})R_S + \frac{2}{K_n})I_{DS} + (V_{GG} - V_{TN})^2 = 0$$

Using Matlab, we have:

```
R1=100;R2=1000e3;R3=550e3;RD=2.5e3;RL=4.7e3;RS=700;Kn=0.1;VTN=2;
RG=1/(1/R2+1/R3);
VB=9;VG=R3/(R2+R3)*VB;Rth=1/(1/R1+1/RG);
p=[RS^2, -(2*(VG-VTN)*RS+2/Kn),(VG-VTN)^2];
Ids=roots(p);
Vds=VB-(RD+RS)*Ids;
Vgs=VG-RS*Ids;
disp(['The two possible values for Ids are     ',num2str(1000*Ids'), ' mA'])
disp(['The two possible values for Vds are     ',num2str(Vds'), ' V'])
disp(['The two possible values for Vgs are     ',num2str(Vgs'), ' V'])
for i=1:2
   if Vgs(i)>VTN
      disp(['The Q-point is IDSQ = ',num2str(Ids(i)*1000),' mA'])
      disp(['                 VDSQ = ',num2str(Vds(i)),' V'])
   end
end
```

```
The two possible values for Ids are     1.9901     1.4609 mA
The two possible values for Vds are     2.6318     4.3252 V
The two possible values for Vgs are     1.8005     2.1709 V
The Q-point is IDSQ = 1.4609 mA
                 VDSQ = 4.3252 V
```

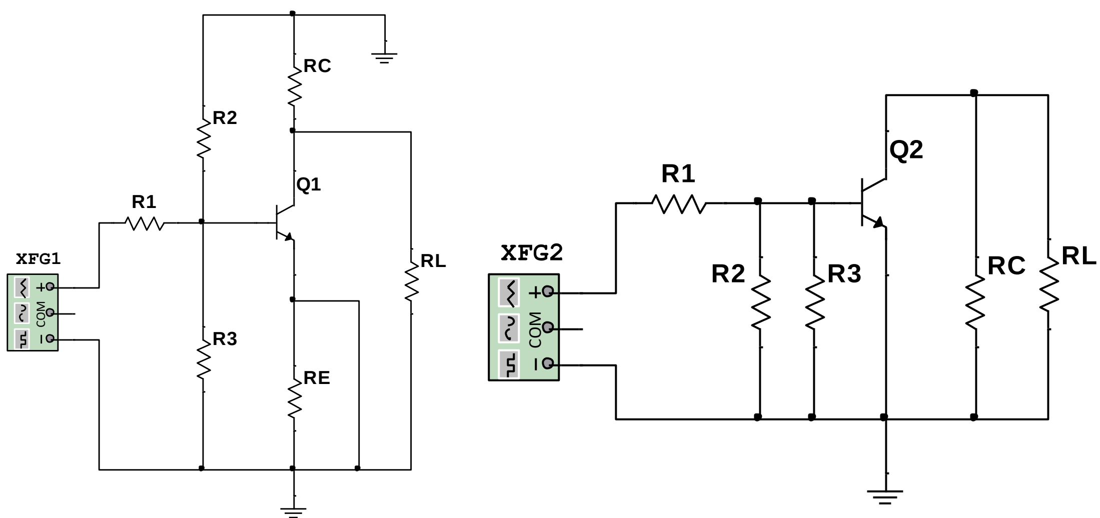
It is clear that the first set of solutions is not valid since $V_{GS} = 1.8005 < V_{TN} = 2V$, negating the saturation region (transistor is OFF).
Hence the solution is the second set of solutions
$I_{DS} = 1.46mA$
$V_{DS} = 4.33\ V$
$V_{GS} = 2.17V$
Since $V_{DS} > V_{GS} – V_{TN} > 0$, the assumption of a saturation region is valid.
These values verify the results obtained in the simulation.

3. AC Equivalent

Let us assume that the capacitors have zero impedance at all frequencies except DC. In addition, all the large capacitors in the power supplies will render the DC power supplies into short circuits from an AC point of view.

3.1 BJT Configuration

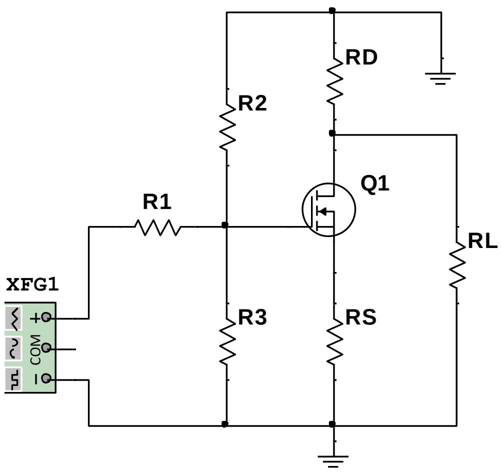
The two versions of the AC equivalent are shown below:

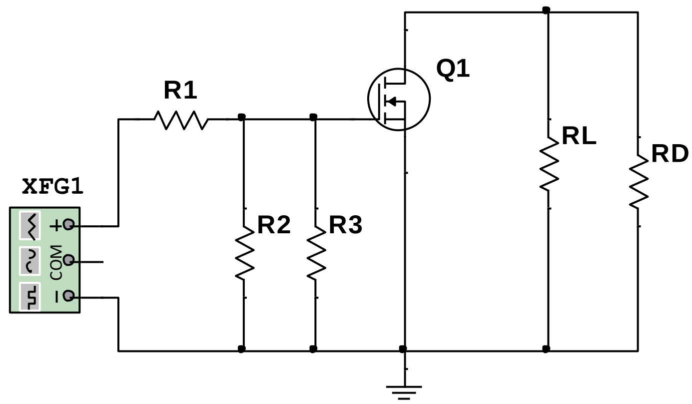


We will complete the AC analysis after we discuss the small-signal equivalent of a transistor.

3.2 MOSFET Equivalent

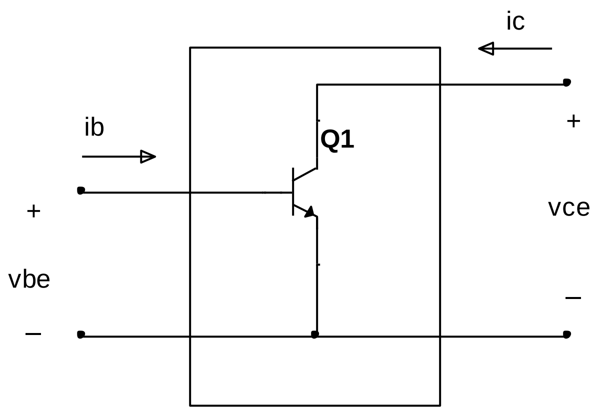
The two circuits shown below are the AC equivalent of the MOSFET-based circuit.





4. Small-Signal Model

4.1 BJT



Using the two-port y-parameter network representation, we have

$$i_b = y_{11} v_{be} + y_{12} v_{ce}$$

$$i_c = y_{21} v_{be} + y_{22} v_{ce}$$

Assuming that in this course that the small-signal common-emitter current gain of the BJT β_0 is such that $\beta_0 = \beta_F$

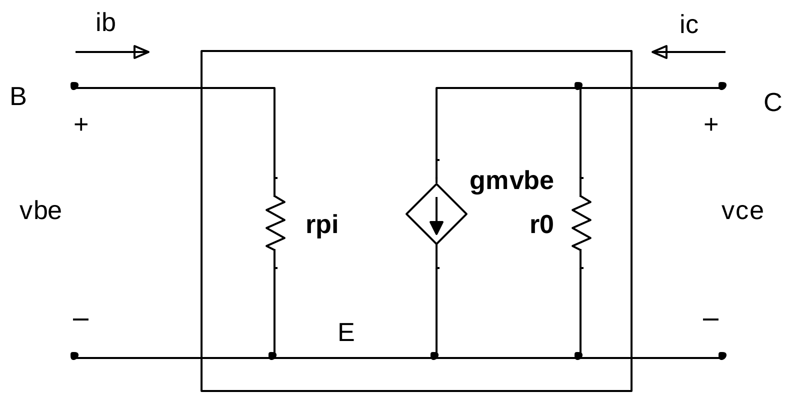
$$y_{11}=\left.\frac{i_b}{v_{be}}\right|_{v_{ce}=0}=\left.\frac{\partial i_B}{\partial v_{BE}}\right|_{Q-point}=\frac{I_C}{\beta_0 V_T}=\frac{40I_C}{\beta_0}=\frac{g_m}{\beta_0}=\frac{1}{r_\pi}$$

$$y_{12}=\left.\frac{i_b}{v_{ce}}\right|_{v_{be}=0}=\left.\frac{\partial i_B}{\partial v_{CE}}\right|_{Q-point}=0$$

$$y_{21}=\left.\frac{i_c}{v_{be}}\right|_{v_{ce}=0}=\left.\frac{\partial i_C}{\partial v_{BE}}\right|_{Q-point}=\frac{I_C}{V_T}=40I_C=g_m$$

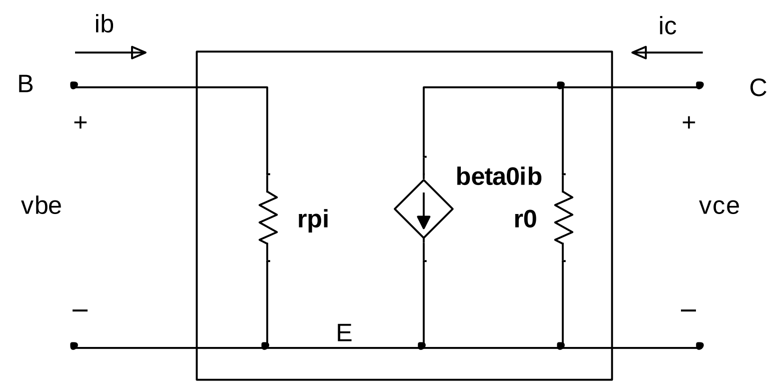
$$y_{22}=\left.\frac{i_c}{v_{ce}}\right|_{v_{be}=0}=\left.\frac{\partial i_C}{\partial v_{CE}}\right|_{Q-point}=\frac{I_C}{V_A+V_{CE}}=\frac{1}{r_0}$$

where I_C and V_{CE} are the values associated with the Q–point, V_A is the Early voltage, and V_T is the Thermal voltage. We see that the y-parameters of the BJT depend on the Q-point. This in turn will affect the characteristics of a BJT-based amplifier.



Since $v_{be} = i_b r_\pi$, hence $g_m v_{be} = g_m\, i_b r_\pi = \beta_0 i_b$

Hence the equivalent circuit that can be useful in DC and AC analyses



$$\text{Total signal} = \text{DC component} + \text{AC component}$$

$$i_C = I_S \exp\left(\frac{v_{BE}}{V_T}\right) = I_S \exp\left(\frac{V_{BE} + v_{be}}{V_T}\right) = I_S \exp\left(\frac{V_{BE}}{V_T}\right) \exp\left(\frac{v_{be}}{V_T}\right)$$

$$= I_C + i_c$$

Note that the Taylor series of $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ...$

Hence

$$i_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left(1 + \frac{v_{be}}{V_T} + \frac{1}{2!}\left(\frac{v_{be}}{V_T}\right)^2 + \frac{1}{3!}\left(\frac{v_{be}}{V_T}\right)^3 + ...\right)$$

Since the DC component is obtained when the AC component is 0

$$i_C = I_C\left(1 + \frac{v_{be}}{V_T} + \frac{1}{2!}\left(\frac{v_{be}}{V_T}\right)^2 + \frac{1}{3!}\left(\frac{v_{be}}{V_T}\right)^3 + ...\right)$$

The transistor will be a linear device when the relationship between the current i_C and the voltage v_{be} is linear (no powers, no cross-products,...)

This will be true when $i_C = I_C\left(1 + \frac{v_{be}}{V_T}\right)$

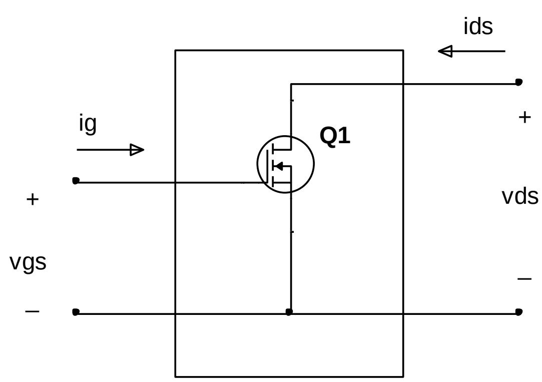
Assuming that the terms of order 3 and higher are negligible, and that the second order term is 10% (0.1) or less of the first term

$$\frac{1}{2!}\left(\frac{v_{be}}{V_T}\right)^2 \le 0.1\frac{v_{be}}{V_T}$$

$$v_{be} \le 0.2V_T = 5mV$$

This value is to be used as a reference, and the designer should decide what is appropriate for the transistor to be considered a linear device.

4.2 MOSFET



Following the same steps as with the BJT, and again using the two-port y-parameter network representation, we have

$$i_g = y_{11}v_{gs} + y_{12}v_{ds}$$

$$i_c = y_{21}v_{gs} + y_{22}v_{ds}$$

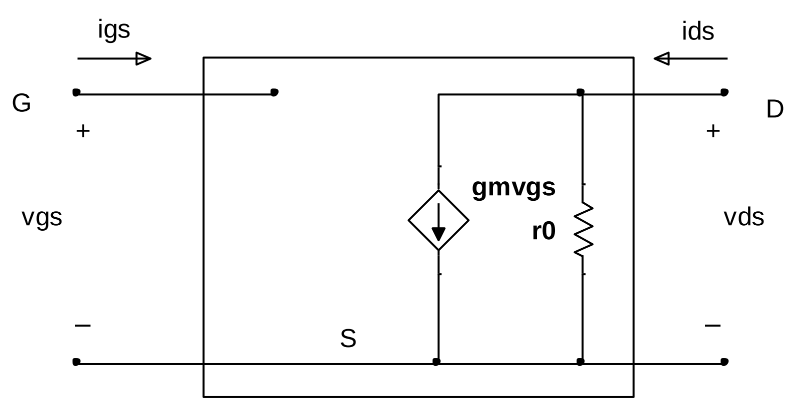
$$y_{11} = \left.\frac{i_g}{v_{gs}}\right|_{v_{ce}=0} = \left.\frac{\partial i_G}{\partial v_{GS}}\right|_{Q-point}$$

$$y_{12} = \left.\frac{i_g}{v_{ds}}\right|_{v_{be}=0} = \left.\frac{\partial i_G}{\partial v_{DS}}\right|_{Q-point} = 0$$

$$y_{21} = \left.\frac{i_{ds}}{v_{gs}}\right|_{v_{ds}=0} = \left.\frac{\partial i_{DS}}{\partial v_{GS}}\right|_{Q-point} = \frac{2I_{DS}}{V_{GS}-V_{TN}} = \sqrt{2K_n I_{DS}} = g_m$$

$$y_{22} = \left.\frac{i_{ds}}{v_{ds}}\right|_{v_{gs}=0} = \left.\frac{\partial i_{DS}}{\partial v_{DS}}\right|_{Q-point} = \frac{I_{DS}}{\frac{1}{\lambda}+V_{DS}} = \frac{1}{r_0}$$

where I_{DS} and V_{DS} are the values associated with the Q–point, λ is the channel-length modulation parameter, and V_{TN} is the Threshold voltage. We see that the y-parameters of the MOSFET again depend on the Q-point. This in turn will affect the characteristics of a MOSFET-based amplifier.



We see that the model for the MOSFET is similar to the model for the BJT except that

$\boldsymbol{\beta_0 = \infty}$**, and** $\boldsymbol{r_\pi = \infty}$**,** but $\boldsymbol{\beta_0 / r_\pi = g_m}$

Total signal = DC component + AC component

$i_{DS} = \frac{K_n}{2}(v_{GS} - V_{TN})^2$ if the MOSFET is in the saturation region

$i_{DS} = I_{DS} + i_{ds} \quad and \quad v_{GS} = V_{GS} + v_{gs}$

$i_{DS} = \frac{K_n}{2}(v_{gs} + V_{GS} - V_{TN})^2$

Hence

$i_{DS} = \frac{K_n}{2}(v_{gs}{}^2 + (V_{GS} - V_{TN})^2 + 2v_{gs}(v_{GS} - V_{TN}))$

$i_{DS} = \frac{K_n}{2}(V_{GS} - V_{TN})^2 + \frac{K_n}{2}(v_{gs}{}^2 + 2v_{gs}(v_{GS} - V_{TN}))$

Since the DC component is obtained when the AC component is 0

$i_{ds} = \frac{K_n}{2}(v_{gs}{}^2 + 2v_{gs}(v_{GS} - V_{TN})) \quad and \quad I_{DS} = \frac{K_n}{2}(V_{GS} - V_{TN})^2$

The transistor will be a linear device when the relationship between the current i_{ds} and the voltage v_{gs} is linear (no powers, no cross-products,...)

This will be true when $i_{ds} = \frac{K_n}{2} 2v_{gs}(v_{GS} - V_{TN})$

Assuming that the second order term is 10% (0.1) or less of the first term

$v_{gs}{}^2 \le 2v_{gs}(V_{GS} - V_{TN})$

$v_{gs} \le 0.2(V_{GS} - V_{TN})$

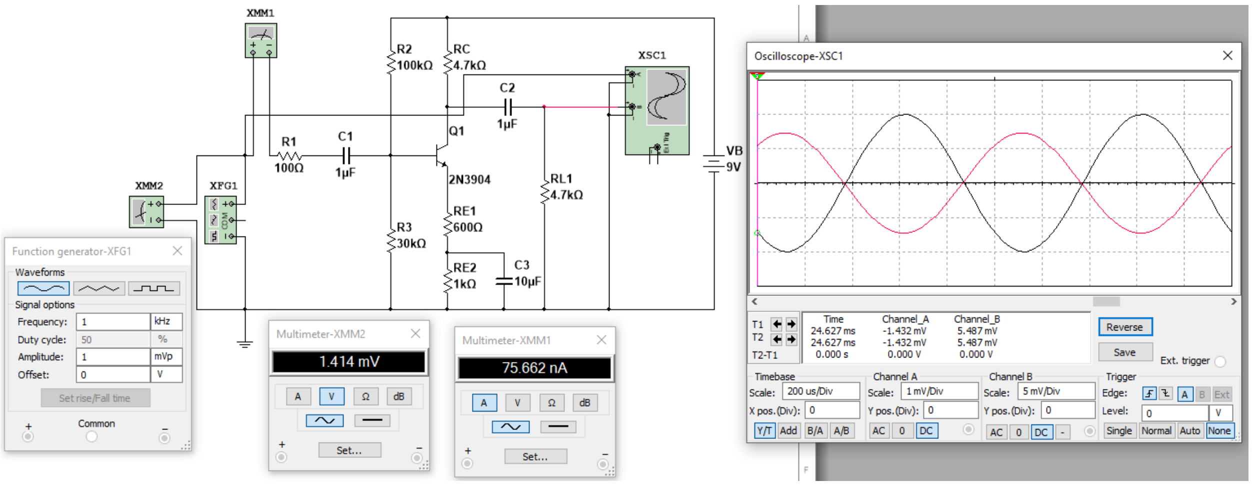
As we see, the MOSFET can handle much larger input voltages than the BJT since $(V_{GS} - V_{TN})$ could be in the fraction of a volt to a volt range.

5. Complete analysis of A common emitter amplifier with emitter degeneration

The circuit shown below is called a common emitter with emitter degeneration because a portion of the emitter resistor is not bypassed.

5.1 DC Analysis

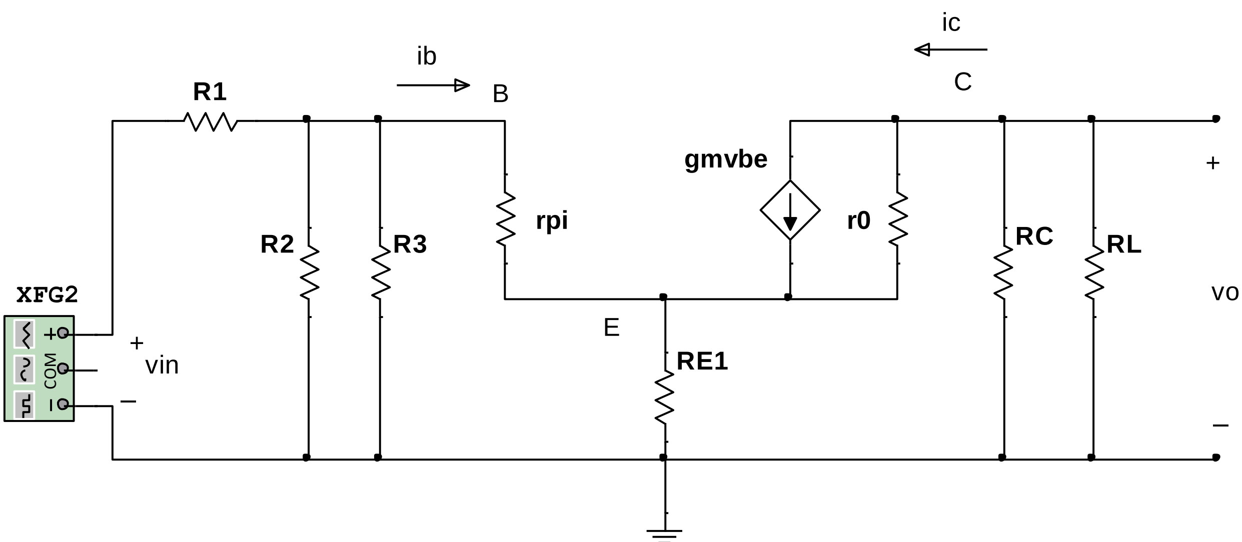
The DC analysis has already been done previously because from the DC point of view (Capacitors open), the circuit is exactly the same as the one in section 2.A
The Q-point is (780μA, 4.09V).
The small-signal parameters are:
$g_m = 40I_C = 40*778\ 10^{-6} = 31.12mS$
$r_\pi = \beta_0/g_m = 136/(31.12\ 10^{-3}) = 4.37k\Omega$
$r_0 = (V_A + V_{CE})/I_C = (74 + 4.09)/(778\ 10^{-6}) = 100.37k\Omega$

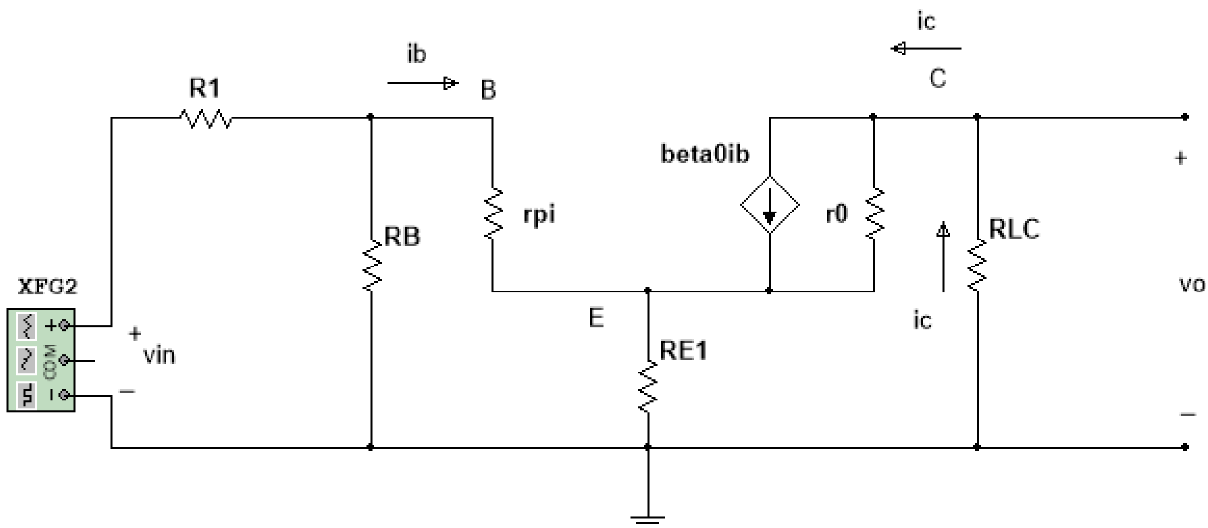


5.2 AC Analysis

5.2.1 Voltage Gain

Let us draw the AC equivalent of the circuit presented in this section. Let us again assume that all the capacitors are equivalent to a short at all frequencies of interest. Let us take the circuit from section 3.1 and replace the transistor by its small-signal equivalent assuming that the input is small enough to keep the transistor operating in the linear region. We then combine some resistors and use the alternate small model for our transistor.





Let $R_B = R_2//R_3$, $R_{LC} = R_L//R_C$, and the current going down r_0 is $i_c - \beta_0 i_b$

$$i_c = -\frac{v_0}{R_{LC}}$$

$$v_b = r_\pi i_b + R_{E1}(i_b + i_c)$$

$$v_b = (r_\pi + R_{E1})i_b + R_{E1}i_c = (r_\pi + R_{E1})i_b - R_{E1}\frac{v_0}{R_{LC}}$$

$$v_0 = r_0(i_c - \beta_0 i_b) + R_{E1}(i_b + i_c)$$

$$v_0 = (r_0 + R_{E1})i_c - (\beta_0 r_0 - R_{E1})i_b = -(r_0 + R_{E1})\frac{v_0}{R_{LC}} - (\beta_0 r_0 - R_{E1})i_b$$

$$i_b = \frac{v_b + R_{E1}\frac{v_0}{R_{LC}}}{(r_\pi + R_{E1})}$$

$$v_0 = -(r_0 + R_{E1})\frac{v_0}{R_{LC}} - (\beta_0 r_0 - R_{E1})\frac{v_b + R_{E1}\frac{v_0}{R_{LC}}}{(r_\pi + R_{E1})}$$

$$v_0 = -(r_0 + R_{E1})\frac{v_0}{R_{LC}} - (\beta_0 r_0 - R_{E1})\frac{R_{E1}\frac{v_0}{R_{LC}}}{(r_\pi + R_{E1})} - (\beta_0 r_0 - R_{E1})\frac{v_b}{(r_\pi + R_{E1})}$$

As a first approximation, $\beta_0 r_0 - R_{E1} \cong \beta_0 r_0$

$$v_0 = -(r_0 + R_{E1})\frac{v_0}{R_{LC}} - (\beta_0 r_0 - R_{E1})\frac{R_{E1}\frac{v_0}{R_{LC}}}{(r_\pi + R_{E1})} - (\beta_0 r_0 - R_{E1})\frac{v_b}{(r_\pi + R_{E1})}$$

$$v_0\left(1 + \frac{r_0 + R_{E1}}{R_{LC}} + (\beta_0 r_0 - R_{E1})\frac{\frac{R_{E1}}{R_{LC}}}{(r_\pi + R_{E1})}\right) = \frac{(\beta_0 r_0 - R_{E1})}{(r_\pi + R_{E1})} v_b$$

$$\frac{v_0}{v_b} = \frac{\frac{(\beta_0 r_0 - R_{E1})}{(r_\pi + R_{E1})}}{1 + \frac{r_0 + R_{E1}}{R_{LC}} + (\beta_0 r_0 - R_{E1})\frac{\frac{R_{E1}}{R_{LC}}}{(r_\pi + R_{E1})}}$$

Multiplying numerator and denominator by R_{LC}

$$\frac{v_0}{v_b} = \frac{R_{LC}\frac{(\beta_0 r_0 - R_{E1})}{(r_\pi + R_{E1})}}{R_{LC} + r_0 + R_{E1} + (\beta_0 r_0 - R_{E1})\frac{R_{E1}}{(r_\pi + R_{E1})}}$$

Multiplying numerator and denominator by $(r_\pi + R_{E1})$

$$\frac{v_0}{v_b} = -\frac{R_{LC}(\beta_0 r_0 - R_{E1})}{(R_{LC} + r_0 + R_{E1})(r_\pi + R_{E1}) + (\beta_0 r_0 - R_{E1}) R_{E1}}$$

Dividing numerator and denominator by $r_0 r_\pi$, and using appropriate approximations

$$\frac{v_0}{v_b} = -\frac{g_m R_{LC}}{1 + g_m R_{E1}}$$

However, we are interested in the voltage gain all the way to v_{in}.

$$\frac{v_o}{v_{in}} = \frac{v_o}{v_b}\frac{v_b}{v_{in}}$$

If we replace the amplifier from the base forward by its input resistance R_{inb}

$$where\, R_{inb} = \frac{v_b}{i_b}$$

Using the previously established equations, we find that

$$R_{inb} = \frac{v_b}{i_b} = r_\pi + R_{E1}\left(1 + \frac{\beta_0 r_0 - R_{E1}}{R_{LC} + r_0 + R_{E1}}\right)$$

If $r_0 \gg R_{LC}$ and R_{E1}, then

$$R_{inb} = r_\pi + \left(\beta_0 + 1\right) R_{E1}$$

Using voltage division

$$v_b = \frac{R_{inb} // R_B}{R_1 + R_{inb} // R_B} v_{in}$$

$$\frac{v_o}{v_{in}} = \frac{v_o}{v_b}\frac{v_b}{v_{in}} = -\frac{g_m R_{LC}}{1 + g_m R_{E1}} \frac{R_{inb} // R_B}{R_1 + R_{inb} // R_B}$$

5.2.2 Input Resistance Seen by v_{in}

$R_{in} = R_1 + R_{inb} // R_B$

5.2.3 Output Resistance

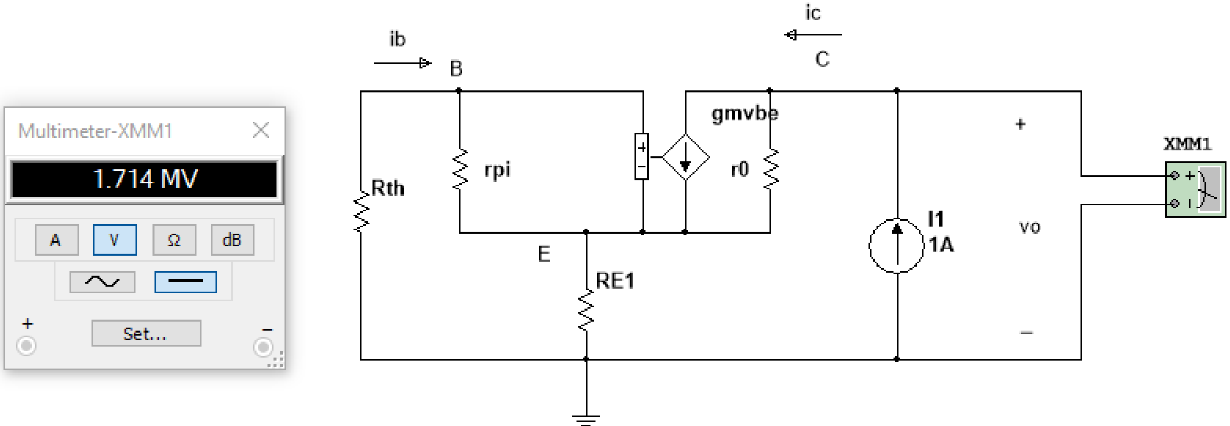
We need to determine the resistance seen from the load R_L when the emitter resistor is not bypassed.

If we replace the voltage source by a short circuit ($v_{in} = 0$), then on the left side, we will have 3 resistors in parallel ($R_1//R_2//R_3$). We will label that combination R_{th}.

$$R_{th} = R_1//R_2//R_3$$

In addition,

To determine the resistance seen from the collector, we remove R_C and R_D, and replace them with a current source whose current is i_x, and find the voltage across it (standard procedure)



Assume the current from the current source is i_x, and the voltage across it is v_x, and we know that $g_m v_{be} = \beta_0 i_b$, and the current flowing through r_0 is $i_x - \beta_0 i_b$,

$$v_x = (i_x - \beta_0 i_b) r_0 + (R_{th} + r_\pi)//R_{E1}\, i_x$$

we also find through current division that

$$i_b = \frac{R_{E1}}{R_{th} + r_\pi + R_{E1}}(-i_x)$$

$$v_x = (i_x - \beta_0 \frac{R_{E1}}{R_{th} + r_\pi + R_{E1}}(-i_x)) r_0 + (R_{th} + r_\pi)//R_{E1}\, i_x$$

Since $R_{ic} = \frac{v_x}{i_x}$,

$$R_{ic} = r_0 \left(1 + \frac{\beta_0 R_{E1}}{R_{th} + r_\pi + R_{E1}}\right) + (R_{th} + r_\pi)//R_{E1}$$

We know that the second term is always negligible compared to the first term

$$R_{ic} \cong r_0 \left(1 + \frac{\beta_0 R_{E1}}{R_{th} + r_\pi + R_{E1}}\right)$$

$$R_{out} = R_C // R_{ic}$$

Example:
Based on the circuit given above, let us determine the voltage gain, the input resistance, and the output resistance, and compare these values to the ones obtained through simulation

```
R1=100;R2=100e3;R3=30e3;RC=4.7e3;RL=4.7e3;RE1=600;RE2=1e3;
RB=1/(1/R2+1/R3);RLC=1/(1/RL+1/RC);
beta=136;VB=9;VBB=R3/(R2+R3)*VB;Rth=1/(1/R1+1/RB);
ICQ=778e-6;VCEQ=4.1;VA=74.03;
gm=40*ICQ; rpi=beta/gm; r0=(VA+VCEQ)/ICQ;
Ric=r0*(1+beta*RE1/(Rth+rpi+RE1));
Gain=-RLC*(beta*r0-RE1)/((RLC+r0+RE1)*(rpi+RE1)+RE1*(beta*r0-RE1));
Gain1=-gm*RLC/(1+RE1/rpi+gm*RE1);
Gain2=-gm*RLC/(1+gm*RE1);
Gain3=-RLC/RE1;
Rinb1=rpi+RE1*(1+(beta*r0-RE1)/(RLC+r0+RE1));
Rinb2=rpi+beta*RE1;
Rin=R1+1/(1/Rinb1+1/RB);
Rout=1/(1/RC+1/Ric);
disp(['The exact voltage gain is ',num2str(Gain)])
disp(['The voltage gain (first approximation) is ',num2str(Gain1)])
disp(['The voltage gain (second approximation) is ',num2str(Gain2)])
disp(['The voltage gain (third approximation) is ',num2str(Gain3)])
disp(['The input resistance is ',num2str(Rin/1000),' kOhms'])
disp(['The output resistance seen from the collector is ',num2str(Ric/1e6),' MOhms'])
disp(['The output resistance seen by RL is ',num2str(Rout/1000),' kOhms'])
```

The exact voltage gain is -3.6856

The voltage gain (first approximation) is -3.6918

The voltage gain (second approximation) is -3.7176

The voltage gain (third approximation) is -3.9167

The input resistance is 18.2145 kOhms

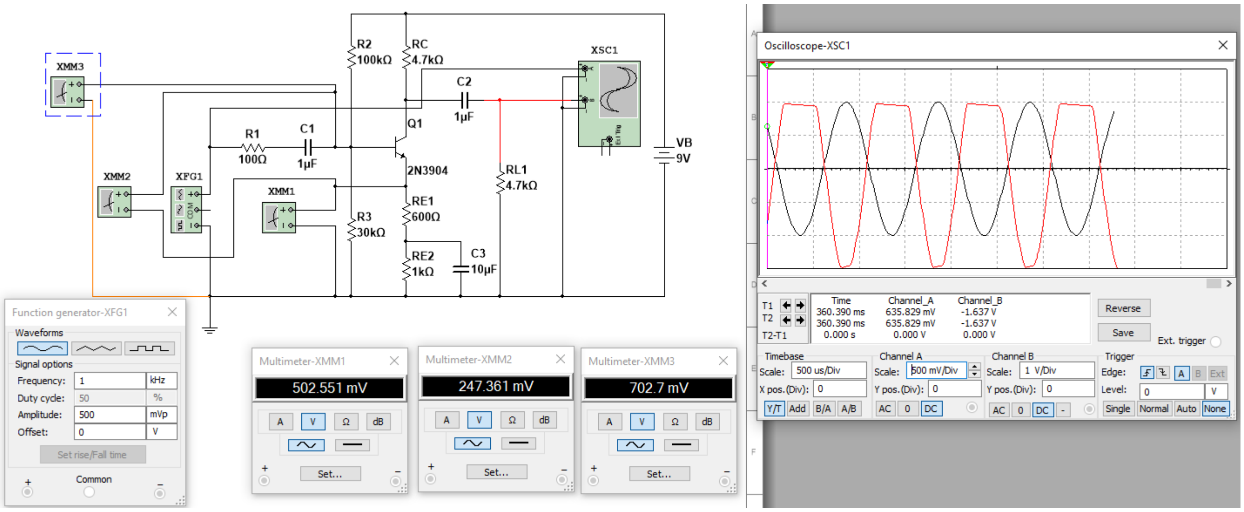
The output resistance seen from the collector is 1.7168 MOhms

The output resistance seen by RL is 4.6872 kOhms

	Calculated	Simulated	Error %
Voltage Gain	-3.69	-3.64	1.36
R_{in}	18.21Ω	18.69kΩ	2.64
R_{ic}	1.72MΩ	1.714MΩ	0.35
R_{out}	4.69kΩ	4.69kΩ	0

We can see that even in the case of the most crude approximation, the presence of R_{E1} allows the gain to be controlled mostly by external components (in this case resistors whose tolerances are at most 1%). In addition, the input resistance is higher because of R_{E1}, and the output resistance is unfortunately high enough not to make this circuit the ideal voltage amplifier.

The simulation may be affected a little bit but the lack of control over the value of the current gain of the transistor. However, the presence of R_{E1} makes it such that there will be a lot less discrepancy in the gap between simulation and practical implementation. In addition, the next figure shows that even though the input exceeds clearly the 5-mV threshold for the B-E junction, it still does not lead to distortion because much of the voltage is dropped across the emitter resistor R_{E1} leaving less than 5 mV across the B-E junction. However, as the input is increased to higher levels, though the B-E emitter junction exceeds the 5-mV threshold, the output waveform seems to the naked eye to be a pure sinewave, but one can see that in this case as we exceed 600 mVp, the distortion becomes visible. The true amount of distortion can only be measured if we had a good spectrum analyzer. With just the naked eye, we can visualize the distortion when it is excessive and we can also note that the RMS value of v_b is not the sum of the RMS values of v_{be} and v_{RE1}. It is evident that the distortion is linked to both the transistor being a nonlinear device and the rails.



6. Complete analysis of A common source amplifier with source degeneration

The same steps will be taken in analyzing a MOSFET-based common-source amplifier. We will be interested again in the voltage gain, the input resistance, and the output resistance. However, the steps are not going to be as extensive since we will benefit from the BJT case since the difference between a BJT and a MOSFET is that

$r_\pi = \infty$

$\beta_0 = \infty$

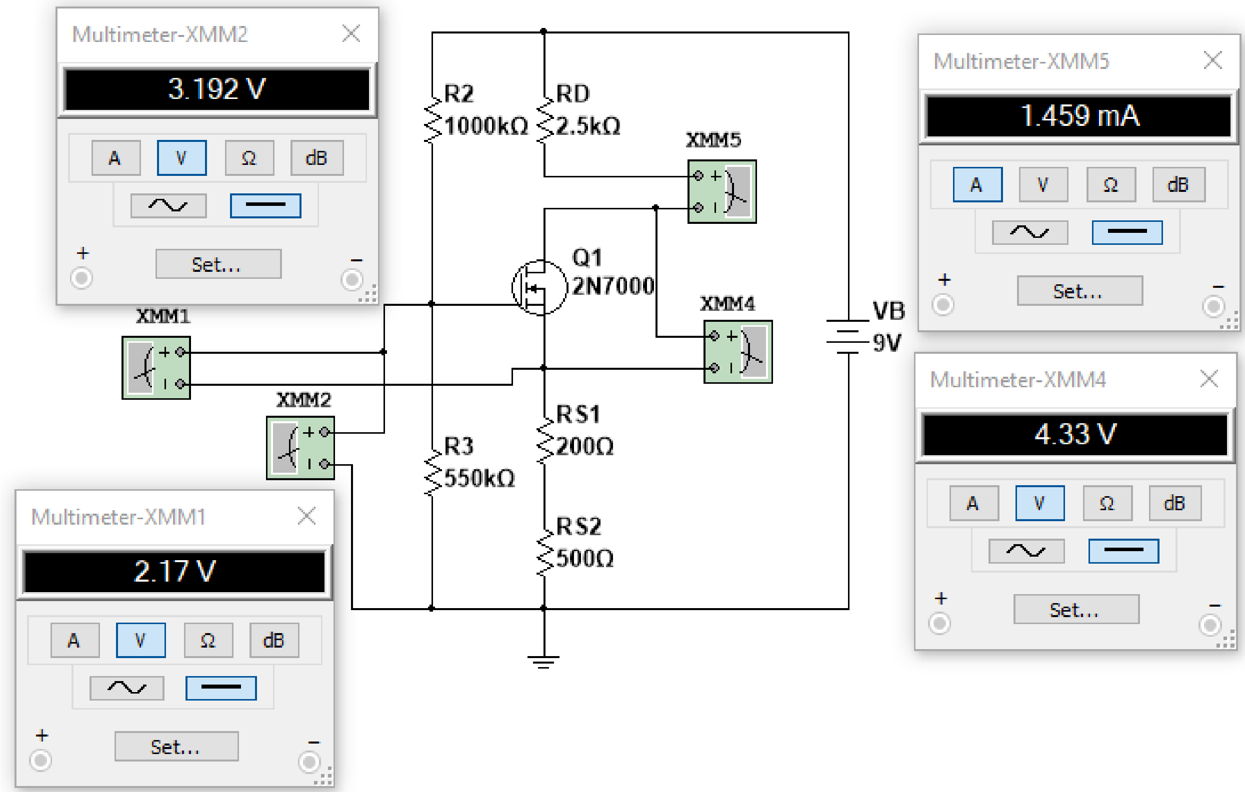
but $\beta_0 / r_\pi = g_m$

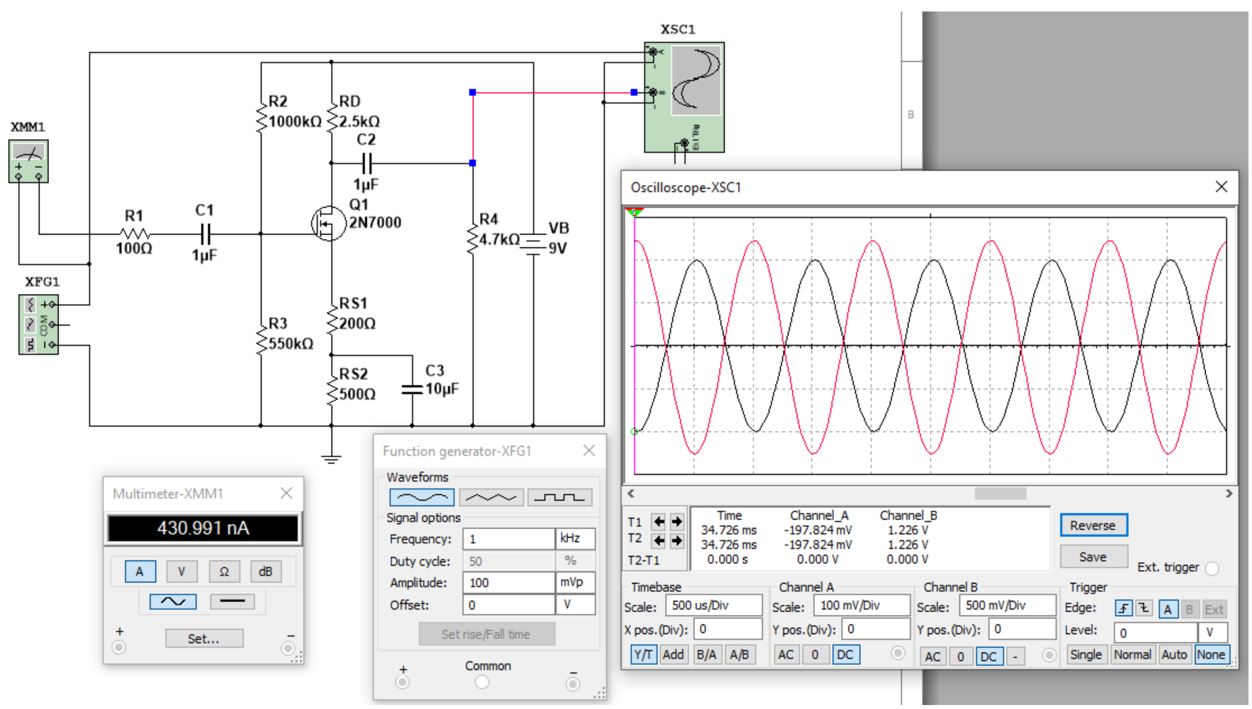
6.1 DC Analysis

The DC analysis has already been done previously because from the DC point of view (Capacitors open), the circuit is exactly the same as the one in section 2.B
The Q-point is (1.46mA, 4.34V).
The small-signal parameters are:

$g_m = (2K_nI_{DS})^{1/2} = (2*0.1*1.46\ 10^{-3})^{1/2} = 17.1\text{mS}$

$r_0 = (1/\lambda + V_{DS})/I_{DS} = (50 + 4.34)/(1.46\ 10^{-3}) = 37.22\text{k}\Omega$

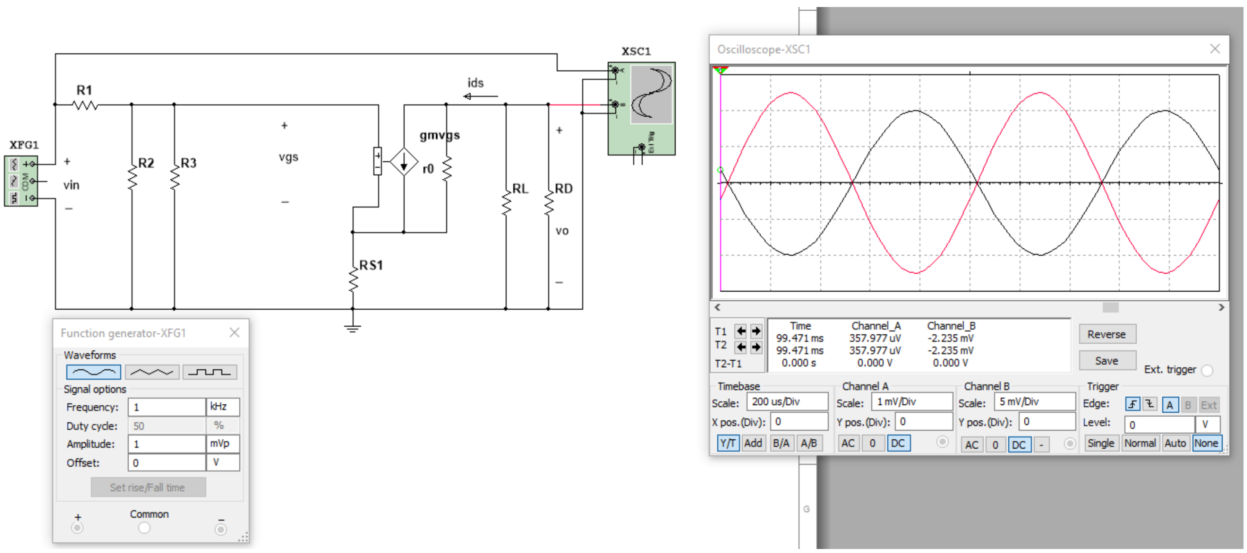

Multimeter-XMM2
3.192 V
R2
1000kΩ
RD
2.5kΩ
XMM5
Multimeter-XMM5
1.459 mA
Q1
2N7000
XMM4
VB
9V
XMM1
XMM2
Multimeter-XMM4
4.33 V
RS1
200Ω
R3
550kΩ
RS2
500Ω
Multimeter-XMM1
2.17 V


XSC1
XMM1
R2
1000kΩ
RD
2.5kΩ
C2
1µF
R1
100Ω
C1
1µF
Q1
2N7000
R4
4.7kΩ
VB
9V
XFG1
RS1
200Ω
R3
550kΩ
RS2
500Ω
C3
10µF
Oscilloscope-XSC1
Multimeter-XMM1
430.991 nA
Function generator-XFG1
Frequency: 1 kHz
Duty cycle: 50 %
Amplitude: 100 mVp
Offset: 0 V
Time
Channel_A
Channel_B
T1 34.726 ms -197.824 mV 1.226 V
T2 34.726 ms -197.824 mV 1.226 V
T2-T1 0.000 s 0.000 V 0.000 V
Timebase Scale: 500 us/Div
Channel A Scale: 100 mV/Div
Channel B Scale: 500 mV/Div

6.2 AC Analysis

6.2.1 Voltage Gain

Let us draw the AC equivalent of the circuit presented in this section. Let us again assume that all the capacitors are equivalent to a short at all frequencies of interest. Let us take the circuit from section 3.1 and replace the transistor by its small-signal equivalent assuming that the input is small enough to keep the transistor operating in the linear region. We then combine some resistors and use the alternate small model for our transistor.



Let $R_G = R_2//R_3$, $R_{LD} = R_L//R_D$, and the current going down r_0 is $i_{ds} - g_m v_{gs}$

$$i_{ds} = -\frac{v_0}{R_{LD}}$$

$$v_g = v_{gs} + R_{S1} i_{ds}$$

$$v_g = v_{gs} - R_{S1} \frac{v_0}{R_{LD}}$$

$$v_0 = r_0 (i_{ds} - g_m v_{gs}) + R_{S1} i_{ds}$$

$$v_0 = (r_0 + R_{S1}) i_{ds} - r_0\, g_m v_{gs}$$

$$v_0 = -(r_0 + R_{S1})\frac{v_0}{R_{LD}} - r_0\, g_m (v_g + R_{S1}\frac{v_0}{R_{LD}})$$

$$v_0 + (r_0 + R_{S1})\frac{v_0}{R_{LD}} + r_0\, g_m R_{S1}\frac{v_0}{R_{LD}} = -r_0\, g_m v_g$$

$$v_0(1 + \frac{(r_0 + R_{S1})}{R_{LD}} + r_0\, g_m \frac{R_{S1}}{R_{LD}}) = -r_0\, g_m v_g$$

$$\frac{v_0}{v_g} = \frac{r_0\, g_m}{1 + \frac{(r_0 + R_{S1})}{R_{LD}} + r_0\, g_m \frac{R_{S1}}{R_{LD}}}$$

$$\frac{v_0}{v_g} = -\frac{R_{LD} r_0\, g_m}{R_{LD} + r_0 + R_{S1} + r_0\, g_m R_{S1}}$$

Exact Expression for the voltage gain

$$\frac{v_0}{v_g} = -\frac{g_m R_{LD}}{1 + \frac{R_{LD} + R_{S1}}{r_0} + g_m R_{S1}}$$

Approximate Expression for the voltage gain if $r_0 \gg R_{LD} + R_{S1}$

$$\frac{v_0}{v_g} = -\frac{g_m R_{LD}}{1 + g_m R_{S1}}$$

$If\ g_m R_{S1} \gg 1$

$$\frac{v_0}{v_g} = -\frac{R_{LD}}{R_{S1}}$$

Using the expressions already derived for the BJT-based amplifier regarding the voltage gain all the way to v_{in}

$$\frac{v_o}{v_{in}} = \frac{v_o}{v_g}\frac{v_g}{v_{in}}$$

In this case

$$R_{ing} = \frac{v_g}{i_g} = \infty$$

Using voltage division

$$v_g = \frac{R_{ing} // R_G}{R_1 + R_{ing} // R_G} v_{in} = \frac{R_G}{R_1 + R_G} v_{in}$$

$$\frac{v_o}{v_{in}} = \frac{v_o}{v_g}\frac{v_g}{v_{in}} = -\frac{g_m R_{LD}}{1 + \frac{R_{LD} + R_{S1}}{r_0} + g_m R_{S1}} \frac{R_G}{R_1 + R_G}$$

as an approximation

$$\frac{v_o}{v_{in}} = -\frac{R_G}{R_1 + R_G}\frac{g_m R_{LD}}{1 + g_m R_{S1}}$$

6.2.2 Input Resistance Seen by v_{in}

$R_{in} = R_1 + R_G$

6.2.3 Output Resistance

Using the expression found for the BJT-based amplifier and remembering that $r_\pi = \infty$, $\beta_0 = \infty$, but $\beta_0 / r_\pi = g_m$

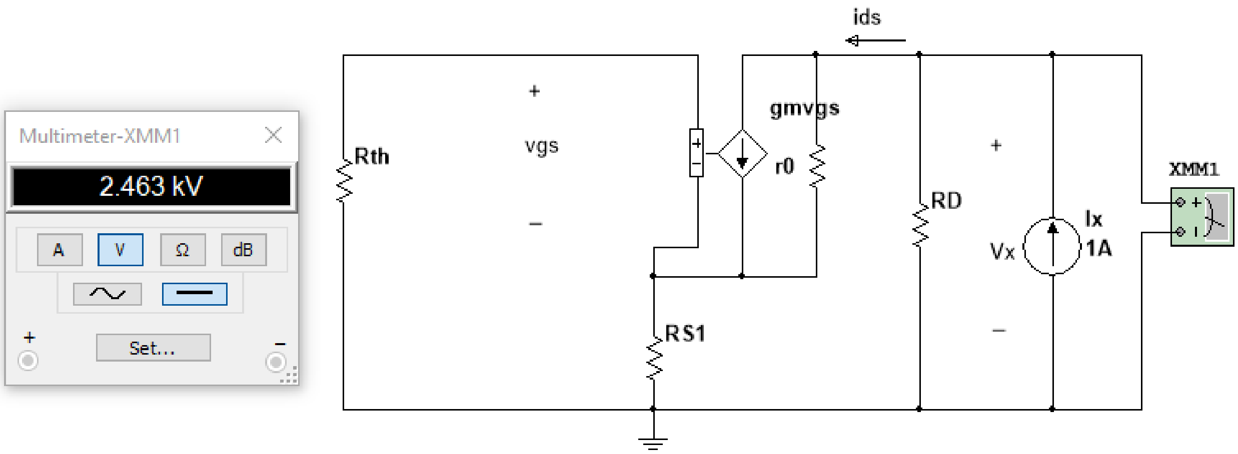
$$R_{id} = r_0\left(1 + \frac{\beta_0 R_{S1}}{R_{th} + r_\pi + R_{S1}}\right) + (R_{th} + r_\pi) // R_{S1}$$

The denominator simplifies to r_π (infinite value)

since $\frac{\beta_0}{r_\pi} = g_m$, then

$$R_{ic} = r_0\left(1 + g_m R_{S1}\right) + R_{S1} \cong r_0\left(1 + g_m R_{S1}\right)$$

$$R_{out} = R_C // R_{ic} \cong R_C$$



Example:
Based on the circuit given above, let us determine the voltage gain, the input resistance, and the output resistance, and compare these values to the ones obtained through simulation

```
R1=100;R2=1000e3;R3=550e3;RD=2.5e3;RL=4.7e3;RS1=200;RS2=500;
RG=1/(1/R2+1/R3);RLD=1/(1/RL+1/RD);
VB=9;VG=R3/(R2+R3)*VB;Rth=1/(1/R1+1/RG);
IDSQ=1.456e-3;VDSQ=4.34;lambda=1/50;Kn=0.1;
gm=sqrt(2*Kn*IDSQ); r0=(1/lambda+VDSQ)/IDSQ;
Rid=r0*(1+gm*RS1);
Gain=-gm*RLD/(1+((RS1+RLD)/r0+gm*RS1));
Gain1=-gm*RLD/(1+gm*RS1);
Gain2=-RLD/RS1;
Ring=Inf;
Rin=R1+1/(1/Ring+1/RG);
Rout=1/(1/RD+1/Rid);
disp(['The exact voltage gain is ',num2str(Gain)])
disp(['The voltage gain (first approximation) is ',num2str(Gain1)])
disp(['The voltage gain (second approximation) is ',num2str(Gain2)])
disp(['The input resistance is ',num2str(Rin/1000),' kOhms'])
```

```
disp(['The output resistance seen from the collector is ',num2str(Rid/1e3),' kOhms'])
disp(['The output resistance seen by RL is ',num2str(Rout/1000),' kOhms'])
```

The exact voltage gain is -6.2412

The voltage gain (first approximation) is -6.3107

The voltage gain (second approximation) is -8.1597

The input resistance is 354.9387 kOhms

The output resistance seen from the collector is 164.6964 kOhms

The output resistance seen by RL is 2.4626 kOhms

	Calculated	Simulated	Error %
Voltage Gain	-6.2412	-6.2	0.64
R_{in}	354.94kΩ	354.46kΩ*	0.14
R_{ic}	164.7kΩ	164.69kΩ	0.006
R_{out}	2.463kΩ	2.463kΩ	0

*R_{in} = 328.6kΩ, as simulated but 354.46kΩ when C_1 was replaced by a short circuit.

Even when C_1 wass increased to much larger values, R_{in} = 328.6kΩ

We can see that in this case, the presence of R_{S1} was such it was not large enough to allow the gain to be controlled mostly by external components (in this case resistors whose tolerances are at most 1%). In addition, we can see that the input resistance is higher because of R_{S1}, and the output resistance is unfortunately high enough not to make this circuit the ideal voltage amplifier.